Sampling rate and aliasing on a virtual laboratory

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Abstract – ***The sampling frequency determines the quality of the analog signal that is converted. Higher sampling frequency achieves better conversion of the analog signals. The minimum sampling frequency required to represent the signal should at least be twice the maximum frequency of the analog signal under test (this is called the Nyquist rate). In the following virtual instrument, an example of sampling is shown. If the sampling frequency is equal or less then twice the frequency of the input signal, a signal of lower frequency is generated from such a process (this is called aliasing).***
The goal of this paper is to teach students basic concepts of sampling rate and aliasing, to become familiar with this concepts.

Keywords: ***sampling, aliasing, virtual instrument.***

I. INTRODUCTION

The analysis of real world signals is a fundamental problem for many engineers and scientists, especially for electrical engineers since almost every real world signal is changed into electrical signals [1].

Sampling is the process of converting an input from a continuous form to a discrete form. In reference to instrumentation this generally means converting analog input (which is continuous in nature) and converting it to digital form (which is discrete in nature). But adequate number of samples must be taken from a given analog signal in order to effectively reconstruct it back from its samples. The 'adequate' number of samples needed is determined by the Nyquist-Shannon theorem. The theorem states that the perfect reconstruction of an analog signal is possible when the sampling frequency is greater than twice the maximum frequency of the signal being sampled [2].

Aliasing is the appearance of phantom frequencies when a signal is not sampled at a high enough rate. In films (which are normally sampled at 24 frames per second) you can often see the wheels of cars or stagecoaches slow down, stand still, or even appear to rotate backwards. This is aliasing [4].

The next figure illustrates how aliasing would occur when the sampling rate is much too low for the frequency of an input signal. The solid curve represents the analog signal at a comparatively high frequency. Circles show where samples were taken at a relatively low sampling rate. The dotted line illustrates the apparent frequency of the sampled waveform, completing about two cycles in the period that the original signal completed 20 cycles.

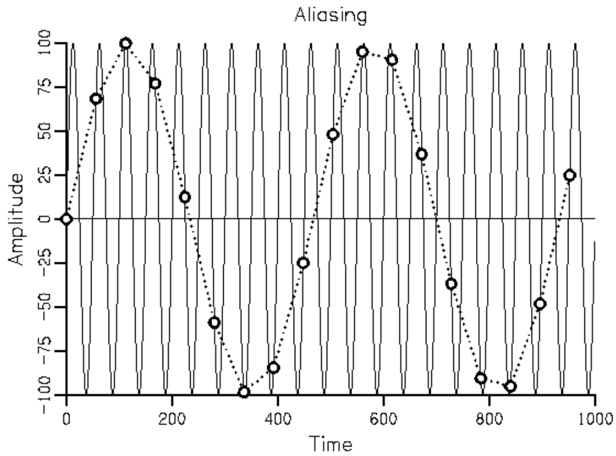


Figure 1. Aliasing occur when the sampling rate is much too low for the frequency of an input signal

For a given input signal of bandwidth f_0, the sampling frequency f_s should be strictly greater than $2f_0$, to ensure perfect reconstruction of the signal from the samples. If $f_s = 2f_0$, then f_s is said to be the Nyquist frequency. It is important to note that information may be lost if a signal is sampled exactly at the Nyquist frequency. For example, the sine wave in Figure 2 has a frequency of 1/2 Hz. The Nyquist frequency is therefore 1 Hz. If we sample the sine wave at a rate of 1 Hz, say at t=0, t=1, t=2, and so on, all the sample values selected will be 0. The signal will look as if it were identically 0, and no reconstruction method will be able to recreate the 1/2 Hz sine wave. This indicates that $f_s > 2f_0$ is a strict condition to be met.

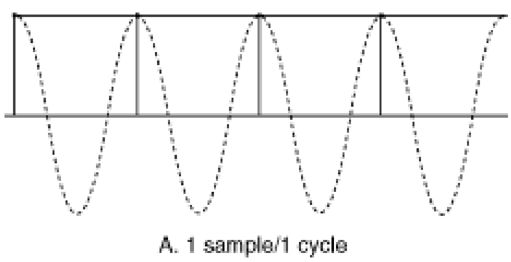


Figure 2. Sampling a sinusoid at Nyquist frequency (Aliased)

As seen in Figure 3, when the sampling frequency is less than the Nyquist rate, the signal is aliased to a frequency less than the original frequency. Perfect reconstruction is observed in Figure 4 where the Nyquist criterion is met. But when sampling frequency is exactly equal to the Nyquist rate the reconstructed waveform appears as an alias at DC.

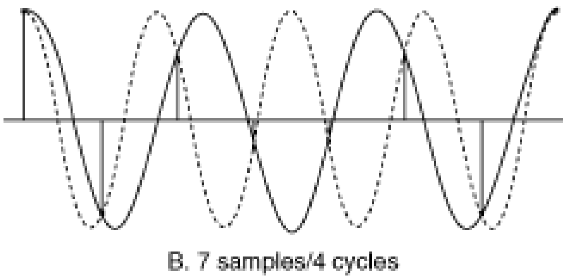


Figure 3. Sampling at fs < 2f

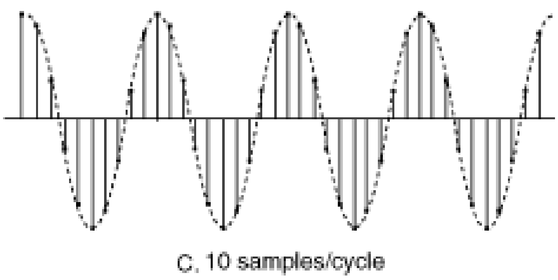
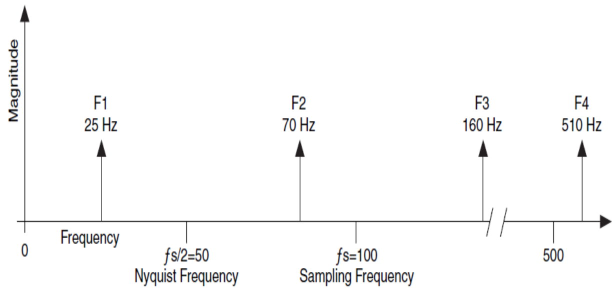


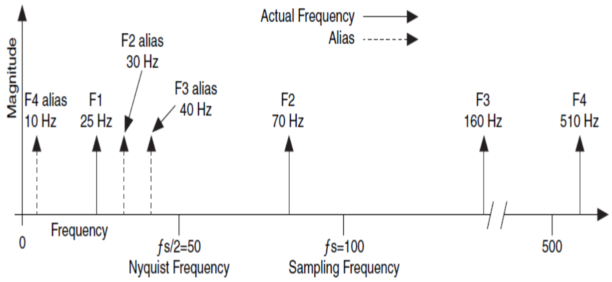
Figure 4. Sampling at fs > 2f

The alias frequency is the absolute value of the difference between the frequency of the input signal and the closest integer multiple of the sampling rate.

For example, assume the sampling frequency, *fs*, is 100 Hz. Also assume that the input signal contains the following frequencies: 25 Hz, 70 Hz, 160 Hz, and 510 Hz, as shown in the following illustration.



Frequencies below the Nyquist frequency (*fs*/2 = 50 Hz) are sampled correctly, as shown in the following illustration. Frequencies above the Nyquist frequency appear as aliases. For example, F1 (25 Hz) appears at the correct frequency, but F2 (70 Hz), F3 (160 Hz), and F4 (510 Hz) have aliases at 30 Hz, 40 Hz, and 10 Hz, respectively.



Use the following equation to calculate the alias frequency:

Alias Freq. = ABS (Closest Int. Mult. of Sampling Freq. – Input Freq.)

where ABS means the absolute value. For example,

$$\text{Alias F2} = |100 - 70| = 30 \text{ Hz}$$

$$\text{Alias F3} = |(2)100 - 160| = 40 \text{ Hz}$$

$$\text{Alias F4} = |(5)100 - 510| = 10 \text{ Hz}$$

To be completely sure that the frequency content of the input signal is limited, a lowpass filter (a filter that passes low frequencies but attenuates the high frequencies) is added before the ADC. This filter is called an anti-alias filter because it prevents the aliasing components from being sampled by attenuating the higher frequencies (greater than Nyquist).

Anti-aliasing filters are analog filters. The following illustration shows an ideal anti-alias filter.

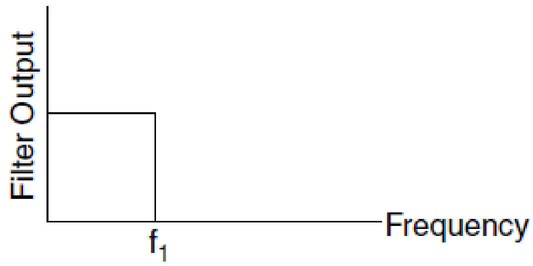


Figure 5. Ideal Anti-alias Filter

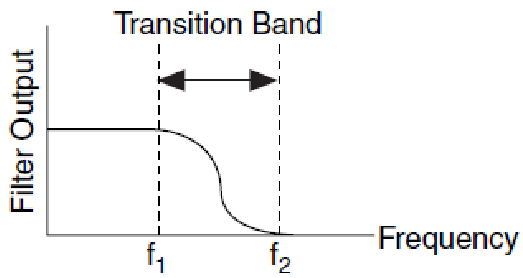


Figure 6. Practical Anti-alias Filter

An ideal anti-aliasing filter passes all the desired input frequencies (below f_1) and cuts off all the undesired frequencies (above f_1). However, an ideal anti-aliasing filter is not physically possible. In practice, filters look as shown in illustration (Figure 6) above. Practical anti-aliasing filters pass all frequencies < f_1 and cut off all frequencies > f_2. The region between f_1 and f_2 is known as the transition band, which contains a gradual attenuation of the input frequencies. Although you want

to pass only signals with frequencies < f_1, the signals in the transition band could still cause aliasing.
Therefore, in practice, you should use a sampling frequency greater than two times the highest frequency in the transition band. Because this sampling frequency turns out to be more than two times the maximum input frequency (f_1), you might see that the sampling rate is more than twice the maximum input frequency.

II. THE VIRTUAL INSTRUMENT

This section presents the virtual instrument programs that were developed to help students for demonstrate the concept of proper and improper sampling, as discussed above.

Figure 7 and 8 present the Front Panel and the Block Diagram of the VI.

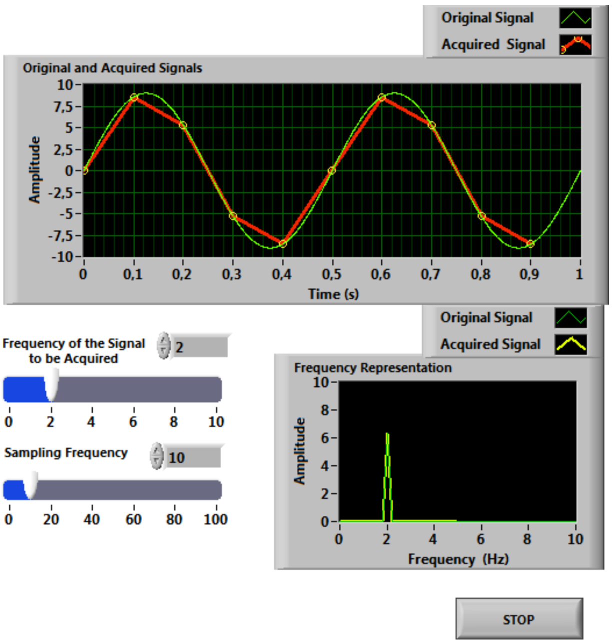


Figure 7.The Front Panel of VI

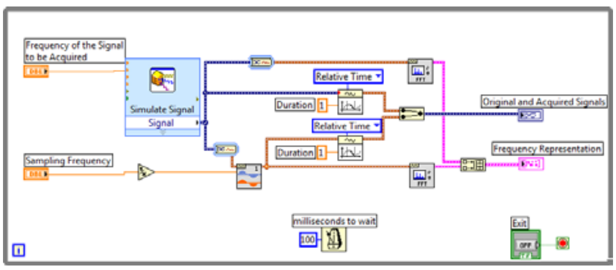


Figure 8. The Block Diagram of VI

The next figures shows examples of a 5 Hz sinusoid sampled at various frequencies. In Figure 9, the sampling frequency is 10 times the analog signal frequency, or 50 Hz. It can be seen that the original signal can be exactly reconstructed from the samples. The signal in Figure 9 has been properly sampled. In, Figure 10 the sampling frequency is 2,5 times the analog signal frequency. This example is more complicated since the signal cannot be exactly reconstructed by simply connecting the sample points. However, the samples actually correspond to only one analog signal since no other sinusoid or combination of sinusoids will produce this pattern. Therefore, the signal in Figure 10 can be said to have proper sampling. In Figure 11, the sampling frequency is 1,25 times the analog frequency. In this example, not only do we have difficulty reconstructing the analog signal, but we have also constructed a different sine wave from the original analog signal. This phenomenon of signals changing frequency after sampling is called **aliasing**. This signal has most certainly been improperly sampled.

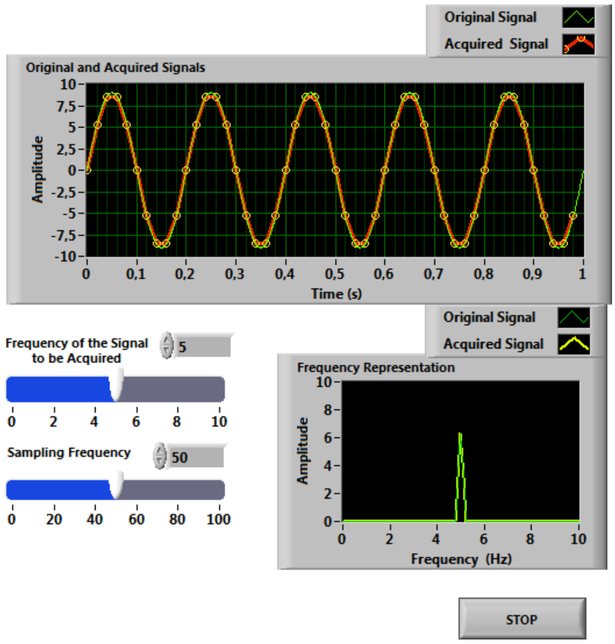


Figure 9. Reconstructing an Analog Signal of 5 Hz. (Sample Frequency = 10 times analog frequency)

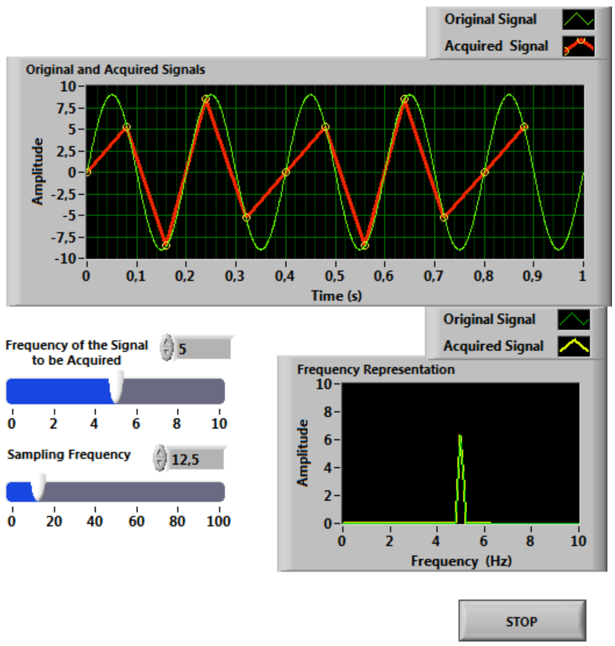


Figure 10. Reconstructing an Analog Signal of 5 Hz. (Sample Frequency = 2,5 times analog frequency)

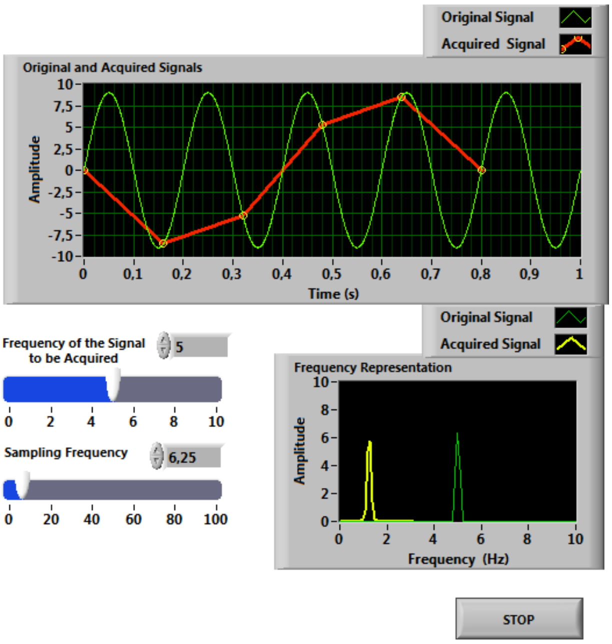


Figure 11. Reconstructing an Analog Signal of 5 Hz. (Sample Frequency = 1,25 times analog frequency)

IV. CONCLUSIONS

As a result of the sampling theorem, a digital signal cannot contain frequencies above one-half the sampling rate or Nyquist frequency. If you are using a sine wave, this is easy, because a sine wave only contains one frequency. However, a square wave contains many higher frequency components in addition to its fundamental repetition frequency. You can see this on the Frequency Spectrum.

If data is taken at a certain sampling rate and the continuous signal frequency is below the Nyquist frequency, the signal can be properly reconstructed from the samples and the frequency of the digitized signal will match the frequency of the continuous signal. However, when the continuous signal frequency is above the Nyquist rate, aliasing changes the frequency into something that can be represented in the sampled data.

Whenever you are sampling, always make sure that:

•The sampling frequency is high enough so that the sampled signal in the computer will be sufficiently true to the original.

•Frequencies at least above theNyquist frequency will be eliminate before sampling, in order to avoid aliasing.

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