# The fundamental interactions of physics derived from the principle of the local gauge symmetry

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***Abstract:*** ***The laws of the electromagnetic interactions were discovered by Maxwell. Later on it was prooved that these laws can be derived also by the principle of the least action if the Lorentz-invariant Lagrangian is invariant also against the local gauge transformations belonging to the U(1) gauge group.***
***This derivation can be considered as a model for the derivation of the laws of the weak and that of the strong interactions using the local gauge transformations belonging to the SU(2) and SU(3) gauge groups, respectively.***

## INTRODUCTION

During the last two centuries the chemists discovered the atoms from the Hydrogene up to the Uranium and a vast multitude of molecules from the Hydrogen molecule up to the Desoxiribo-nucleonicacid and beyond. During the last century the physicists discovered the elementary particles which are the smallest building blocks of the atomic and subatomic systems. At the moment we are convinced that the number of the elementary particles is 12. They can be devided into two groups, namely into the groups of leptons and quarks.

| LEPTONS | | QUARKS | |
|---|---|---|---|
| e electron | e neutrino | d down | u up |
| μ muon | μ neutrino | s strange | c charm |
| τ tau | τ neutrino | b bottom | t top |

All the 12 elementary particles are fermions having spin 1/2 and they follow the Pauli-principle. Because of tis reason they are considered to be the building stones of matter, since a well defined quantum state can not be occupied by more than a single fermion.

Among normal conditions the quarks can not exist in free state, they exist only in bound states: three qarks together, in bound state (q1,q2,q3) form a baryon (for example a proton, or a neutron, or a hyperon, etc.)
One quark and an antiquark together, in a bound state (q1, q~2) form a meson, (for example a pion, or a kaon, or a charmonium, etc.)
Step by step it was discovered that there are three fundamental interactions among the particles which are responsible for their beheviour, both in free and in bound states.
These are the electromagnetic interaction (acting among particles having electric charge, or magnetic moment), the weak interaction (acting among all of the 12 elementary particles), and
the strong interaction (acting among the quarks).
In the nature there exists also a fourth fundamental interaction, that is the gravitation. Its effect , however, is negligible in the realm of the microparticles.

## THE ELECTROMAGNETIC INTERACTION

The Dirac–equation for the elementary fermions i. e. for leptons and for quarks having rest mass m can be written in the following form:

$$\left(i\gamma^j \partial_j - m\right)\Psi = 0\,,$$
$$\overline{\Psi}\left(i\gamma^j \partial_j + m\right) = 0\,,$$

where the 4*4 Dirac matrices $\gamma^j$ are defined by the anticommutators

$$\gamma^j\gamma^k + \gamma^k\gamma^j = 2g^{jk}, \; (j,k = 0,1,2,3),$$

$$g_{jk} = g^{jk} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

The solutions of the Dirac–equation can be obtained in terms of plane waves:

$$\Psi(x) = u(p)e^{-ipx}, \; u = \begin{pmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \end{pmatrix},$$

where $px = p_j x^j = p^0x^0 - p^1x^1 - p^2x^2 - p^3x^3$

or,

$px = Et - \vec{p}\vec{x}$.

The amplitude u(p) is the solution of the algebraic equation

$$(\gamma^j p_j - m)u(p) = 0$$

which exists only if the relativistic relations among the mass m, the energy E and the momentum $\vec{p}$ are fullfilled:

$$E = +sqrt(m^2 + \vec{p}^2), \text{ or}$$
$$E = -sqrt(m^2 + \vec{p}^2).$$

It can be proved that these solutions are eigenstates of the spin operator $s_z$ with the eigenvalues +1/2, or –1/2.

It is easy to see that the Dirac–equation is invariant with respect of the global gauge transformation:

$$\Psi'(x) = e^{-i\Theta(x)}\Psi(x),$$

where Θ is a real constant.

Now we introduce a very important requirement, namely we demand that the Dirac–equation must be invariant against the local gauge transformation:

$$\Psi'(x) = e^{-i\Theta(x)}\Psi(x),$$

where $\Theta(x)$ is a real number, depending on the local coordinates $x^0$, $x^1$, $x^2$ and $x^3$.

The original Dirac–equation does not fullfill this requirement, however if we introduce an extra term of the form $e\gamma^j A_j(x)e$, it is easy to see that the “new” Dirac–equation

$$\left(\gamma_j\left[i\partial^j - eA^j(x)\right] - m\right)\Psi(x) = 0$$

is transformed into

$$\left(\gamma_j\left[i\partial^j - eA'^j(x)\right] - m\right)\Psi'(x) = 0$$

if we perform the local gauge transformation defined by

$$\Psi'(x) = e^{-i\Theta(x)}\Psi(x),$$
$$A'_j(x) = A_j(x) + \frac{1}{e}\partial_j\Theta(x).$$

In order to obtain information about A*j*(x), let us construct the Lagrangian of the field A*j*(x), then apply the principle of the least action:

$$\delta s = 0,$$

where s is the action integral defined as

$$s = \int d^4x \; \mathcal{L}\left(A^j(x), \partial^j A^k(x)\right)$$

The Lagrangian-density $\mathcal{L}\left(A^j(x)\partial^j A^k(x)\right)$ can be constructed from Lorentzian scalars:

$$\mathcal{L} = M^2 A_j A^j + bF_{jk}F^{jk} + zG_{jk}G^{jk}$$

where $F_{jk}(x) = \partial_j A_k(x) - \partial_k A_j(x)$,

$$G_{jk}(x) = \partial_j A_k(x) + \partial_k A_j(x).$$

The local gauge invariance required for the Dirac–equation must be required for the complete Lagrangian density as well. This means that the coefficients $M^2$ and z must vanish: $M^2=0$ and z=0.

Consequently the Lagrangian density invariant against both the Lorentz transformation and the local gauge transformation have the following simple form:

$$-\frac{1}{4}F_{jk}(x)F^{jk}(x).$$

Finally the complete Lagrangian can be written as follows:

$$\mathcal{L} = \overline{\Psi}\left(i\gamma^{j}\partial_{j} - m\right)\Psi$$
$$-\frac{1}{4}F_{jk}F^{jk}$$
$$-\overline{\Psi}e\gamma_{j}A^{j}\Psi \ .$$

The first term describes the free fermion field $\Psi(x)$ the second the free vector field $A_j(x)$ and the last term corresponds to the interaction of the fermion field $\Psi(x)$ and the vector field $A_j(x)$.

By the help of the variational calculus we can obtain the necessary conditions of the minimum of the action integral s:

$$\left[\gamma_{j}\left(i\partial^{j} - eA^{j}(x)\right) - m\right]\Psi(x) = 0 \ ,$$
$$\overline{\Psi}(x)\left[\gamma_{j}\left(i\partial^{j} + eA^{j}(x)\right) + m\right] = 0 \ ,$$
$$\partial_{j}F^{jk}(x) = j^{k}(x) \text{, where}$$
$$j^{k}(x) = e\overline{\Psi}(x)\gamma^{k}\Psi(x) .$$

We have derived the field equations of the electrodynamics i. e. the Maxwell-equations.

It is obvious that $A_j(x)$ is identical with the vector potential, $j^k(x)$ is identical with the electric current density and $F_{jk}(x)$ is identical with the field strength tensor.

From the gauge symmetry we have got as a by product the condition $M^2=0$. In the quantumelectro-dynamics the physical meaning of M is the mass of the photon.

This means that from the Lorentz- symmetry and from the local gauge symmetry we have obtained not only the field equations of the electrodynamics but also the most important property of the photon. We have got the whole optics!

## CONCLUSION

This derivation of the laws of the electromagnetic interaction can serve as a model for the derivation of the laws of other interactions e. g. for the weak and for the strong interactions.

Even if we were blind we would be able to discover the light by the help of the principle of the local gauge invariance. As a matter of fact we are blind in respect of the weak and the strong interactions.

By the help of the generalizations of the local gauge transformation we are able to discover completely the theory of the weak and that of the strong interactions.

In the case of the electromagnetic interaction the local gauge transformations form a particular group, namely the group of the 1 dimensional, unitarian matrices. Its conventional name is U(1). It is associated with a single real field $A_k(x)$.

In the case of the weak interaction the transfor-mations form an other particular group, namely the group of the unitarian, 2 dimensional, special matrices. Its conventional name is SU(2). It is associated with the three real parameters W1(x), W2(x) and W3(x).

In the case of the strong interaction the transfor-mations form again a particular group, namely the group of the unitarian, 3 dimensional, special matrices. Its conventional name is SU(3). It is associated with eight real parameters G1(x), G2(x), G3(x), G4(x), G5(x), G6(x), G7(x) and G8(x).

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