Design of Linear Parametric Internal Model Controller-Polynomial approach

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***Abstract*—This paper presents design of linear parametric model of the Internal Model Controller. Internal model controller is designed using conventional method and it is parameterized using linear parameterization with Hurwitz polynomials. Model reference parameterization is used to obtain a reference model. The modified structure of the linear parametric internal model controller is drawn and it is presented.**

***Index Terms*— parameterization, internal model Controller, Robust Control.**

I. INTRODUCTION

Robust controllers are very popular in now days because of its best performance and less maintenance. In commercial point of view, robust controllers are lucrative. Internal model principle is rapidly growing in robust controller design in the recent years.

Internal Model Controller (IMC) is a one of the robust controller proposed based on internal model principle. It is very successfully applied in domestic and industrial application because of its easy implementation and requires less parameter tuning also IMC controller is the best alternative for PI controllers.

The important key issues concerning IMC are speed response, low pass filler and internal model. The robust performance of this controller is based on these parameters. Parametric model controller design is the one of the best method to improve the robust recital of the controller because of the matrix approach is involved to improve the performance of the controller. Some parameters in this controllers depends on several others parameters and their relationship is same.

Modified Internal Model Controller (IMC) is proposed in paper [1]. Parametric approach robust controllers are presented in [2]. Further, the direct model reference adaptive IMC is presented in [3] [4].

In this paper, IMC controller is designed and it is parameterized. From the parameterized equation controller block diagram is drawn. To the best of authors' knowledge, this approach is made possible first time in IMC. The proposed method is very general and different versions of it are possible.

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II. CONTROLLER DESIGN

To design the parametric model, one of the parameter in the controller is the subset of the parametric family of the controller. It is also called finite dimensional parametric model. The filtering operation in these controllers associated with parameter. Distribution of parameters in the parametric model is finite dimensional.

A. Internal Model Controller

Fig.1 shows the general IMC approach. Internal model principle is implemented and explained in [5]. The IMC design takes into account the model uncertainties and it allows straight forward relation of controller settings with the model parameters. In the first order system, IMC controller, which is insensitive to time delay and parameter deviation, and it is approximately equal to the PI controller [6]. The response of IMC is sluggish although it does not have any overshoot [7] and integral action of this controller is used to eliminate offset. The closed loop transfer function of IMC is

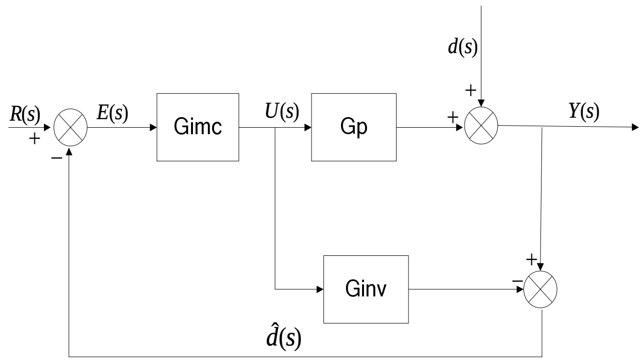

Fig. 1. General IMC diagram

$$Y(s)=\frac{G_{imc}(s)G_{p}(s)R(s)+[1\text{-}G_{imc}(s)G_{inv}(s)]d(s)}{1+[Gp(s)\text{-}G_{inv}(s)]G_{imc}(s)} \quad (1)$$

B. Controller Design

The IMC controller design method for first order system is prescribed in [7].

First order plant model $G_p = \dfrac{K_P\,(S-a)}{(S+b)}$ (2)

Internal model $G_{inv} = \dfrac{S+b}{S-a}$ (3)

Low pass filter F is used to avoid model mismatch

$$F = \frac{1}{(1+fs)^n} \tag{4}$$

Where f is speed response tuning parameter and n is order of low pass filter

$$H = FG_{inv} = \frac{S+b}{(S-a)(1+fS)^n} \tag{5}$$

The controller is

$$G_{imc} = \frac{H}{1-HG_{inv}} = \frac{(S+b)(S-a)}{(1+fS)^n(S-a)^2-(S+b)^2} \tag{6}$$

IMC is stable in low frequencies because of the low pass filter used in the controller.

C. Pararmetrization

Parameterization methods described in [9] is used to obtain parametric internal model controller. Consider the polynomial equation given below,

$$G_{imc} = \frac{S^2+bS-aS-ab}{a^2-b^2+(nfa^2-2a-2b)S+\left(\frac{n(n-1)}{2!}f^2a^2-2nfa\right)S^2+\mathbf{L}+f^nS^{n+2}} \tag{7}$$

The initial condition is zero

$$Z(S) = S^2 + S(b-a) - ab \tag{8}$$

$$R(S) = a^2-b^2+\left(nfa^2-2a-2b\right)S+\left(a^2\frac{n(n-1)}{2!}f-2anf\right)S^2+\mathbf{L}+f^nS^{n+2} \tag{9}$$

The transfer function is

$$U(S) = \frac{Z(S)}{R(S)}E(S) \tag{10}$$

The parametric vector,

$$q^* = \left[\left(f^n,\cdots,nf-2a-2b,a^2-b^2\right),\left(1,(b-a),-ab\right)\right]^T \tag{11}$$

The dimension of the parametric vector is less than the number of functions of the parametric vector. Derivatives of the input signal vector is

$$U(S)' = \left[\left(E(S)^2,E(S)\right),\left(-U(S)^{n+2},-U(S)^{n+1},\cdots,-U(S)\right)\right]^T \tag{12}$$

$$U(S)' = \left[a_{n-1}^T(S)E(S),-a_{n+2}^T(S)U(S)\right] \tag{13}$$

Where

$$a_i(S) @ \left[S^i,S^{i-1},\cdots,1\right]^T \tag{14}$$

The compact form the equation (10) & (11) is

$$U(S)^{(n)} = q^*U(S)' \tag{15}$$

To avoid the differentiated signals $U(S)^{(n)}$ & an n^th^ order stable filter $\frac{1}{\Lambda(S)}$ is included in the both side of the equation (15).

Where $\Lambda(S)$ is Hurwitz of arbitrary degree $r>0$,has the same input properties of equation(10)

Where

$$Z(S) = \frac{1}{\Lambda(S)}U(S)^{(n)} = \frac{S^n}{\Lambda(S)}U(S) \tag{16}$$

$$f(S) = \left[\frac{a_{n-1}^T(S)}{\Lambda(S)}E(S),\frac{-a_{n+2}^T(S)}{\Lambda(S)}U(S)\right]^T \tag{17}$$

and $\Lambda(S) = S^n + l_{n-1}S^{n-1} + \cdots + l_0$ (18)

$$Z(S) = q^{*T}f(S) \tag{19}$$

$Z(S)$ is a scalar signal and $f(S)$ is a vector signal, generated without differentiation.

Now $\Lambda(S) = S^n + l^Ta_{n-1}(S)$ (20)

$$l = \left[l_{n-1},l_{n-2},\cdots,l_0\right] \tag{21}$$

Then $Z(S) = \frac{S^n}{\Lambda(S)}U(S)$ (22)

Substitute S^n from equation (20) in equation (22)

$$Z(S) = \frac{\Lambda(S)-l^Ta_{n-1}(S)}{\Lambda(S)}U(S) \tag{23}$$

$$Z(S) = U(S) - l^T\frac{a_{n-1}(S)}{\Lambda(S)}U(S) \tag{24}$$

$$\therefore U(S) = Z(S) + l^T\frac{a_{n-1}(S)}{\Lambda(S)}U(S) \tag{25}$$

$$Z(S) = q^{*T}f(S) = q_1^{*T}f_1(S) + q_2^{*T}f_2(S) \tag{26}$$

Where

$$q_1^{*T} = \left[f^n,\cdots,nf-2a-2b,a^2-b^2\right]^T \tag{27}$$

$$q_2^{*T} = \left[1,(b-a),-ab\right]^T \tag{28}$$

$$f_1(S) = \frac{-a_{n+2}(S)}{\Lambda(S)}U(S) \tag{29}$$

$$f_2(S) = \frac{a_{n-1}(S)}{\Lambda(S)}E(S) \tag{30}$$

$$y(S) = q_1^{*T}f_1(S) + q_2^{*T}f_2(S) - l^Tf_2(S) \tag{31}$$

$$y(S) = q_l^{*T}f(S) \tag{32}$$

Where $q_l^{*T} = [q_1^{*T},q_2^{*T}-l^T]^T$ (33)

Consider equation (32) and include the identity (34)

$$W_m(S)W_m^{-1}(S) = 1 \tag{34}$$

$W_m(S) = \frac{Z_m(S)}{R_m(S)}$ is a transfer function with degree one, and $Z_m(S)$ and $R_m(S)$ are Hurwitz polynomials.

q_l^* is a constant vector. Then equation (32) becomes

$$y(S) = W_m(S)q_l^{*T}W_m^{-1}(S)f(S) \tag{35}$$

Let $y(S) = \frac{1}{W_m(S)}f(S)$ (36)

From equation (13)

$$y(S)=\left[\frac{a^{T}{}_{n-1}(S)}{W_m(S)\Lambda(S)}E(S),\frac{-a^{T}{}_{n+2}(S)}{W_m(S)\Lambda(S)}U(S)\right]^T \tag{37}$$

$$y(S)=W_m(S)q_l{}^{*T}y(S) \tag{38}$$

$\frac{a^{T}{}_{n-1}(S)}{W_m(S)\Lambda(S)}$ are proper transfer function with stable poles.

$$y(S)=\left[y_1(S)^T,y_2(S)^T\right]^T, \tag{39}$$

Where

$$y_1(S)=\frac{a^{T}{}_{n-1}(S)}{W_m(S)\Lambda(S)}E(S) \tag{40}$$

$$y_2(S)=\frac{-a^{T}{}_{n+2}(S)}{W_m(S)\Lambda(S)}U(S) \tag{41}$$

The dimension of $y(S)$ & $Z_m(S)$ is arbitrary and dimension of $y(S)$ depends on the order n of $\Lambda(S)$ and $Z_m(S)$.

D. Model Reference Pararmetrization

The reference model is derived based on [10]. Form equation (10) it is used to track the desired trajectory and to minimize the plant error. This is a direct reference model design.

$$\left(a^2-b^2+\left(nfa^2-2a-2b\right)S+\left(a^2\frac{n(n-1)}{2!}f-2anf\right)S^2+.......f^nS^{n+2}\right)u(S) \tag{42}$$
$$=\left(S^2+S(a-b)-ab\right)E(S)$$

$$a=0,b=0 \tag{43}$$

$$U(s)=\frac{E(s)}{(1+fS)^n-1} \tag{44}$$

The reference model is strictly proper plant

$$W_m(S)=\frac{1}{(1+fS)^n-1} \tag{45}$$

$$W_m(S)=\frac{1/f^nS}{S^{n-1}+l_0} \tag{46}$$

$$l_0=\frac{n}{f^{n-1}} \tag{47}$$

$(S^{n-1}+l_0)$ is a factor of $\Lambda(S)$ i.e

$$\Lambda(S)=(S^{n-1}+l_0)\Lambda_q(S) \tag{48}$$

$$\Lambda_q(S)=S^{n-2}+q_{n-2}S^{n-3}+\cdots+q_1S+1 \tag{49}$$

E. Parametric Internal Model Controller

From equation (32), (38), (39)

$$u(S)=\left[W_m(S)[q_1^{*T},q_2^{*T}-l^T]^T\left[y_1(S)^T,y_2(S)^T\right]^T\right]E(S) \tag{50}$$

Fig.2 shows the designed parametric model of the controller, modification of internal model controller is shown clearly in the controller side with the internal model.Linearized parametric reference model is include in the controller to increase the parameter uncertainty. The main advantage of this controller is it has a capability to regenerate the parameter so it acts as a parameter insensitive controller. This model is also known as series parallel model. The transient response and the steady state value will not affect the closed loop pole.

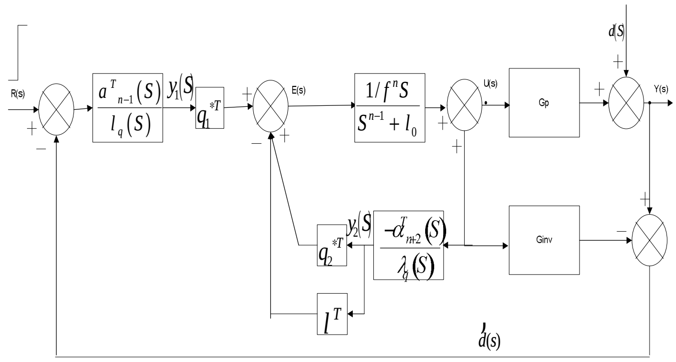

Fig: 2. Linear parametric internal model controller

III. CONCLUSIONS

Internal model controller is designed conventional method. Linear parametric polynomial approach is used to improved structure of the internal model bounded parametric controller id designed and it is presented. Improvements in linear parametric Internal Model Controller are shown and it is well suited for practical application. The mathematical design procedure is presented. Further investigation is in progress to validate the results with conventional controllers. This controller is possible to use unmodeled part of the plant and bounded disturbances.

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BIOGRAPHIES

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