Determination of electricity nodal prices using Lagrange method

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Abstract - With the functioning of the electric power market all participants in the market sell and buy electric power on the united equilibrium price. United equilibrium price for all participants of market can't be established to be upon consideration of system limitations. Their individual equilibrium prices, called nodal price, correspond to different generator and load units. In the work is ilustrated possibility of the nodal prices calculation for load nods with the aid of the indeterminate Lagrange multipliers based on the example of the electric power system test scheme. The corresponding optimization problem of the linear programming preliminarily is solved for the revealing of active limitations to the variables.

Keywords : electric power, optimization, nodal price, active and passive limitations, Lagrange multipliers

I. INTRODUCTION

The ongoing structural rebuilding of the power industry in the world leads up to forming the power markets [1-4]. The final cause of inclusion the new economical relations is reducing of energy charge, through the realization of pricing in market-guided medium. Reformation of the power industry prescribes a passage to price forming of a competitive device at electric energy market. In return for traditional unitary board of industry administrating, built on the adequate reflection of price signal subjects of market incorporation. On the first place are going out economical factors, which are defining the interests of market participant relations, by certainly execution of demands for reliability and electric power quality.

The main idea of competition in electric power industry in covered in division of production transmission, i.e. electric power industry as a product, from commercial point of view is detached from transmission as a kind of service. Through the concluded agreements or participation in market, consumer establishes a seller, price, sales conditions. In the sphere of transmission and distribution competition is eliminated: here are operating natural monopolies. For providing an effective competition are claimed serious excessive strength, as well as appropriate electric net conduction.

The increase of competitive participants of power market, from one side, is providing en route of dividing big companies- monopolists by spheres of business and creating independent companies in the sector of generation, transmission, distribution and energy sales. Though is possible another way- by drawing in to the market new independent manufacturers.

Competition enforces participants to lower price for their products and services, lowering costs for production involving in new technologies to the production process, to be marketable at the market.

An observance of power supply safety factor has anitial sense at the forming the competition medium. It is inadmissible for sellers to ignore safety questions due of commercial aims at competition markets of energy. In this case, talk is about modern and qualitative equipment repairing, maintenance of the power reserves, coordinating electric power stations development, electricity and heat supply networks, obviation of transmission lines overloading.

The most important part of new relationships organization between power system and consumers is development of methods and algorithms for define comepetitive prices for power energy. It is necessary at every stage- palnning, exploitation and conducting of power industry development. In the world practice there isn't any united access to solution of this problem. Today are known some kinds of accesses to the modelling of competitive electric power prices [5-7]. An algorithm of establishing the prices for electric power in the load bus with using an uncertain Langrange multipliers is reviewed below.

II. OPTIMIZATION TASK OF COMPETITIVE MARKET

In classical staging the task of optimizing electric power system (EPS) regime, is comprising of minimization the summary costs $C(P,Q)$ on the generation of real (P) and reactive (Q) power (production of electric power) at noticing totality of technological restrictions. These restrictions are made by the way of equalities and inequalities. At the complete structure the limitations are divided at node limitations and line limitations. To the node limitations belongs the limitations for generator nodes:

- at real and reactive powers

$$P_{gi}^{\min} \le P_{gi} \le P_{gi}^{\max},\ Q_{gi}^{\min} \le Q_{gi} \le Q_{gi}^{\max}; \qquad (1)$$

- at voltage modulus

$$U_{gi}^{\min} \le U_{gi} \le U_{gi}^{\max}. \qquad (2)$$

To the line limitation belongs limitations at the load flows and line currents:

$$P_l \le P_l^{adm}\ ,\ I_l \le I_l^{adm}\ , \tag{3}$$

as well as limitations at the changes of transformation coefficients

$$k_l^{\min} \le k_l \le k_l^{\max}\ . \tag{4}$$

The limitations in the shape of equalities are made by the balance of power in the net

$$\sum_i P_{gi} - \sum_j P_{dj} - \sum_l \Delta P_l = 0 \tag{5}$$

and water expenditure balance at the hydroelectric power stadions (HES)

$$\sum_{t=1}^{T} W_{HESi}^{t} - W_{HESi} = 0\ , \tag{6}$$

where W_{HESi}^{t} - water expenditure in an hour t at i HES; W_{HESi} - the setted recycled amount of water at the time interval T.

Summarizing at (5) is made up at all generator nodes (i), consumption nodes (j) and lines (l).

At the transition to competitive relations in power industry, optimizing task of the electricity regime is inessentially transforming. Changes are concerning to the objective function. The solution of optimizing task of competitive market, as in the case of plan economic, is possible only by using technically-economic model, describing functional links of economical characteristics and EPS parametrs. The above mentioned limitations stays without changes. Without observation of the system limitations and technical losses entailed with transmission of electric power, solution of the optimization task is the objective function maximization

$$F = \sum_{j \in D} c_{dj} \cdot W_{dj} - \sum_{i \in G} c_{gi} \cdot W_{gi}\ , \tag{7}$$

where c_{dj}, c_{gi} - consumer and generator price announcements; W_{dj}, W_{gi} - subjects announced amounts of electricity consumption and generation in time; D, G - the number of load bus and generator nodes.

Function (7) is known as market participants welfare function. It is important to mark, that by conducting the market (according to (7)) preference is given to market participants, who indicates to the smallest prices. This gives the minimization of consumption to maintain the constant balance between load and generation.

Enlisting, that an time interval *Δt* for determination the amount of electric power is equal both for consumers and sellers (ususually 1 hour), objective function (7) can be modified to a mode

$$F = \left(\sum_{j \in D} c_{dj} \cdot P_{dj} - \sum_{i \in G} c_{gi} \cdot P_{gi} \right) \to \max\ . \tag{8}$$

For the inelastic market, where is satistied predictable demand for an electric power, the optimized task of competitive market leads to minimizing of function

$$F = \sum_{i \in G} c_{gi} \cdot P_{gi}\ . \tag{9}$$

Optimization of reviewed objective functions must be realized with accounting of sooner reviewed limitations (1)-(6).

In the absence the calculation of system limitations and technical losses, connected with the electric transmission, the search for optimal solution is most simple, because united equilibrium price will be formed in all units of electrical network. The latter means that all participants in the market sell and buy electric power on this price.

With technological limitation being, which are affecting on the price formation, in different nodes are forming different nodal prices. Determination of nodal price, as a rule, is realized through finding the objective resourses of established evaluation to the assumed limitations- variables, which can be calculated using uncertain Lagrange multipliers.

III. THE DEFINITION OF NODAL PRICES

For all EPS always exists two basic factors, which are affecting on the price formation. To these factors are relating system limitations and technical losses. Variety of system limitations builds up net limitations(3) and technical limitations to the generator work regime (1). Amid the limitation can be dealed out actives and passives. If in some point P^* one or another parameter assumes the border value, then appropriate limitation is called active, but the other limitations- passives. If prematurely is known, which of limitations(1) and (3) are passives, and which are active, then first of them can be eliminated from the survey, but second notice as a limitation in a form of equality. Below we will take note, that active limitations can be revealed in the process of solving the optimization tasks.

For simplification of further reflections we can admit, that power losses are included in summarized electric power load P_D ($P_D = \sum_j P_{dj}$). And in electric power model in lines will be reviewed middle value of power flow.

The optimization task of competitive market with accounting of limitations(1)-(3) is a typical task of linear programming (LP) because the objective function(7) is linear. Solving this task, without detailed revision of LP alghoritms, can be executed using office software of modern computers. For this purpose are used programms-solvers, that are meant for optimizing the solving of different tasks (linear and nonlinear). The solving of LP task with the help of solvers allows to take out active limitations. In the attitude with the mathematical programming theory these limitations can be transfered to the form of equality. Thereby, the exception of passive limitations in optimized task allows to use the Lagrange method to define the nodal prices. We will use follows uncertain Lagrange multipliers: λ - variable, wich determines "closing" demanded price claim of generation for the power balance(5); μ_L - variables, which reflect the revealed active limitations to the power flows; ν_p - variables, which reflect the revealed active limitations to the power of the generator (marginal values max or min are designated as P_{gj}^{adm}). Active limitations for the generation and the flows according to the connections can be written down in the form of equalities

$$P_{gj}^{adm} - P_{gj} = 0,\ j = \overline{1, G_p};\ P_l^{adm} - P_l = 0,\ l = \overline{1, L_l}, \quad (10)$$

where G_p, L_l - quantity of generators and lines, which are on the marginal (permissible) values..

Taking into account the conventional signs minimization of objective function (9) can be carried out by Lagrange method. Lagrange function in this case can be recorded as

$$L = \sum_{i \in G} c_{gi} \cdot P_{gi} + \lambda\left(P_D - \sum_{i \in G} P_{gi}\right) + \sum_{l=1}^{L_l} \mu_l \left(P_l^{att} - P_l\right) + \\ + \sum_{j=1}^{G_p} \nu_j \left(P_{gj}^{att} - P_{gj}\right) \quad (11)$$

Power flux in the line l can be recorded through the current distribution coefficients α_{li} and the nodes power $P_i = P_{gi} - P_{di}$ in the form.

$$P_l = \sum_{i+1}^{n} \alpha_{li} \cdot P_i ,$$

where n - quantity of independent nodes in EPS.

Thus the Lagrange function of the competitive market optimization task can be generated in the form of

$$L = \mathrm{c}^T \cdot \mathrm{P}_g + \lambda\left(P_D - \mathrm{e}^T \mathrm{P}_g\right) + \mu_L^T \left(\mathrm{P}_l^{att} - \alpha \cdot \mathrm{P}\right) + \\ + \nu_p^T \left(\mathrm{P}_g^{att} - \mathrm{P}_g\right), \quad (12)$$

where λ, μ, ν - indefinite Langrange multipliers; $\boldsymbol{P}$ - vector of power nodes (without balance node); $\boldsymbol{P}_g^{att}$ - vector of revealed limiting (max or min) values of the real power generation; ***T***- symbol of transposition.

Differentiation of functions (12) on all variables - generated powers and uncertain Lagrange multipliers leads to a system of linear equations

$$\frac{\partial L}{\partial \boldsymbol{P}_g} = 0, \ \frac{\partial L}{\partial \lambda} = 0, \ \frac{\partial L}{\partial \boldsymbol{\mu}_L} = 0, \ \frac{\partial L}{\partial \boldsymbol{\nu}_p} = 0. \quad (13)$$

The physical meaning of calculated uncertain Lagrange multipliers is following. Multiplier λ describes the movement value of the objective function as a result of small demand change in particular node. It corresponds to nodal prices cover the load demand of the cheaEPSt demanded generator in the absence of regime restrictions. In this case, all elements of vector $\boldsymbol{\mu}_L$ will be equal to 0, and elements of the vector $\boldsymbol{\nu}_p$ will be equal +1 when given generator unit reach top limit of limitation on the power, and - 1, if below the lower limit of limitation on the power.

Value λ reflects the cost of « closing » (last) 1 MW coverage of demand from the power grid generators. Calculated cost is the minimum possible, taking into account all the restrictions imposed on the production and transfer of power in EPS. Lagrange multipliers for the restrictions on controllable connections and external crossflows $\boldsymbol{\mu}_L$ from the economic point of view, interpreted as the price of the last 1 MW throughput of sections. They describe the change value of objective function as a result of small change in the throughput of certain section. For example, at small increase in throughput of section there is a possibility to transfer more power by this section from cheaper generators and, accordingly, to unload more expensive, that allows to improve the value of objective function.

In the presence of restrictions vector elements $\boldsymbol{\mu}_L$, corresponding to connection number (line, section), are different from 0. Prices in the remaining load nodes in this case defined by the expression

$$\boldsymbol{c}_n = \lambda \cdot \boldsymbol{e} + \boldsymbol{\nu}_p + \boldsymbol{\alpha}^T \cdot \boldsymbol{\mu}_L , \quad (14)$$

where ***e*** - unit vector (each coordinate is equal 1 - $e_i = 1$).

IV. EXAMPLE

Consider the pricing process on example of real EPS (Fig. 1). Powers of generators and their pricing application are given in tab.1, which showing at what floor price generator (as a subject of the market) is ready to sell the electric power. As can be seen from the EPS scheme total consumer load P_D is equal to 500 MW. Let us assume that the power losses are included in the load.

Tab.1

Nr	*1*	*2*	*3*	*4*
$P_{gj}^{\max}$, MW	*200*	*200*	*200*	*200*
$P_{gj}^{\min}$, MW	*100*	*100*	*100*	*80*
c_{gi}, €/MWh	*45*	*100*	*80*	*60*

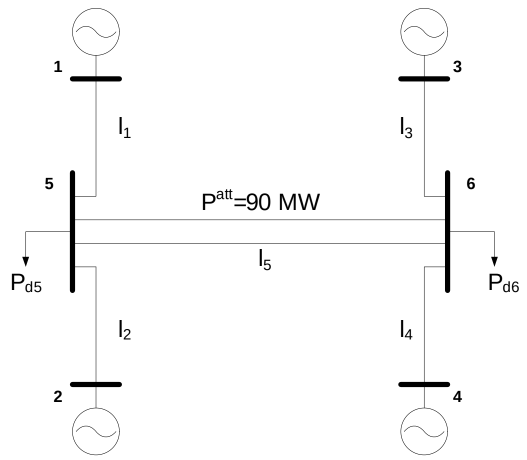

Fig. 1. EPS scheme

Construction of supply step curve and the determination of the point of its intersection with the line of total demand determine equilibrium price (fig. 2). In this case, it is equally $c_* = 80$ €/MWh. It means that all market participants under this price sell and buy electricity. Besides, this solution has defined also structure of the working generating equipment. In our case the second generator, which falls out of the market, does not get to number of sellers. Applications of remaining generators are satisfied in the following volume: $P_{g1} = 200$ MW, $P_{g3} = 100$ MW, $P_{g4} = 200$ MW.

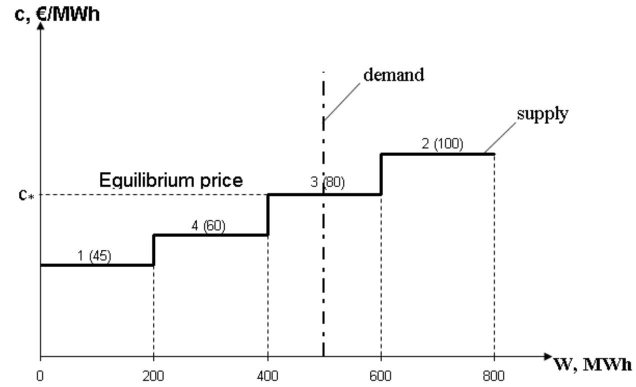

Fig.2. Equilibrium price determination on the auction results in the EPS

Objective function of optimization problem (7) will be relevant

$$F = 45 \cdot 200 + 80 \cdot 100 + 60 \cdot 200 = 29000 .$$

Analysis of the auction results without restriction on the throughput of the communication line between the left and right groups of generators and consumers shows reloading line l_5. At throughput of a line $P_{l5}^{att} = 90$ MW, in accordance with the planned regime, it must be transferred 200 MW power, that is 110 MW more than the allowable value. Hence the system operator (SO) must decide on necessity of mode correction that as a result can change both equipment structure, and loading of generators. With regard to the mode correction unified equilibrium price cannot be established for manufacturers of the electric power and its consumers. To maintain the balance of power in the whole on EPS necessity to include in the market the generator with the most expensive electricity could be an objective fact. Accounting of the given circumstance requires a consideration of the so-called nodal prices, which may be different for the loading nodes.

Fulfill the task with full equipment of EPS and all the constraints by simplex method LP. Initial target function represented as

$$F = 45 \cdot P_{g1} + 100 \cdot P_{g2} + 80 \cdot P_{g3} + 60 \cdot P_{g4} + c_5 P'_{g5} + c_6 P'_{g6} ,$$

where P'_{g5} and P'_{g6} are additional variables that describe the load nodes 5 and 6. They can be interpreted as a possible generation of load nodes. In pending scheme of EPS generated sources allow to provide fully all customers without their switching-off. Price-based applications c_5 and c_6 can be regarded as infinitely high.

The power balance in EPS is written as

$$\sum_{i=1}^{4} P_{gi} + P'_{g5} + P'_{g6} = P_{d5} + P_{d6} = 500 .$$

Bilateral limitation of the form (1) using data from Table 1 represent as a pair of inequalities ($P_{gi} \ge P_{gi}^{\min}; P_{gi} \le P_{gi}^{\max}$). For our data obtain:

$$\begin{aligned}
P_{g1} &\ge 100,\\
-P_{g1} &\ge -200,\\
P_{g2} &\ge 100,\\
-P_{g2} &\ge -200,\\
P_{g3} &\ge 100,\\
-P_{g3} &\ge -200,\\
P_{g4} &\ge 80,\\
-P_{g4} &\ge -200,\\
P'_{g5} &\ge 0,\\
P'_{g6} &\ge 0.
\end{aligned}$$

The power flow P_{l5} in the line l_5 can be expressed on the basis of matrix elements of current distribution coefficients of the EPS scheme. Matrix of current distribution coefficients calculated concerning on the node balance 1 has the form

$$\begin{array}{c} \\ l_1 \\ l_2 \\ \boldsymbol{\alpha} = l_3 \\ l_4 \\ l_5 \end{array} \begin{array}{c} \begin{matrix} n_1 & n_2 & n_3 & n_4 & n_5 & n_6 \end{matrix} \\ \begin{pmatrix} 0 & -1 & -1 & -1 & -1 & -1 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{pmatrix} \end{array} .$$

Thus, the expression of the flow through line l_5 can be written as $P_{l5} = P_{g3} + P_{g4} + P'_{g6} - 100$.

Given the throughput restriction of this line ($P_{l5}^{att} = 90$ MW) receive the restriction of type (3) in the form $-P_{g3} - P_{g4} \ge -190$, because $P'_{g6} = 0$.

The simplex method implementation of considered problem LP leads to following optimal parameters of the regime: $P_{g1} = 200$ MW, $P_{g2} = 110$ MW, $P_{g3} = 100$ MW, $P_{g4} = 90$ MW.

The value of objective function equally

$$F = 45 \cdot 200 + 100 \cdot 110 + 80 \cdot 100 + 60 \cdot 90 = 33400$$

The received result has allowed to reveal three active restrictions among inequalities of a solved problem. In the future, these restrictions could be replaced by equalities of type (10). Together with the equation of power balance, these equalities form a system of linear equalities

$$\left.\begin{aligned} \sum_{i=1}^{4} P_{gi} + P'_{g5} + P'_{g6} &= 500 ,\\ P_{g1} &= 200 ,\\ P_{g3} &= 100 ,\\ P_{g3} + P_{g4} + P'_{g6} &= 190 . \end{aligned}\right\}.$$

The last equality corresponds to the electric mode of EPS at transmitted power maximum through line l_5.

Define the unknown Lagrange multipliers involved in the nodal prices establishment of load nodes. Lagrange function of structure (12) in our case can be written as

$$L = 45\cdot P_{g1} + 100\cdot P_{g2} + 80\cdot P_{g3} + 60\cdot P_{g4} + \lambda\left(500 - \sum_{i=1}^{4} P_{gi} - P'_{g5} - P'_{g6}\right) +$$
$$+ \nu_1\left(200 - P_{g1}\right) + \nu_3\left(200 - P_{g3}\right) + \mu_5\left(190 - P_{g3} - P_{g4} - P'_{g6}\right)$$

After differentiation we will receive two groups of linear equations:

$$\left.\begin{aligned} \frac{\partial L}{\partial P_{g1}} &= 45 - \lambda - \nu_1 = 0\\ \frac{\partial L}{\partial P_{g2}} &= 100 - \lambda = 0\\ \frac{\partial L}{\partial P_{g3}} &= 80 - \lambda - \nu_3 - \mu_{l5} = 0\\ \frac{\partial L}{\partial P_{g4}} &= 60 - \lambda - \mu_{l5} = 0\\ \frac{\partial L}{\partial P'_{g5}} &= c_5 - \lambda = 0\\ \frac{\partial L}{\partial P'_{g6}} &= c_6 - \lambda - \mu_{l5} = 0 \end{aligned}\right\}(A), \qquad \left.\begin{aligned} \frac{\partial L}{\partial \lambda} &= 500 - \sum_{i=1}^{4} P_{gi} - P'_{g5} - P'_{g6} = 0\\ \frac{\partial L}{\partial \nu_1} &= 200 - P_{g1} = 0\\ \frac{\partial L}{\partial \nu_3} &= 100 - P_{g3} = 0\\ \frac{\partial L}{\partial \mu_{l5}} &= 190 - P_{g3} - P_{g4} - P'_{g6} = 0 \end{aligned}\right\}(B).$$

Vectors $\boldsymbol{\nu}_p$ and $\boldsymbol{\mu}_L$, respectively, for all nodes and lines of EPS in transposed form can be represented as

$\nu_p^T = (\nu_1,\ 0,\ \nu_3,\ 0,\ 0,\ 0)$,

$\mu_L^T = (0,\ 0,\ 0,\ 0,\ 0,\ \mu_5)$.

The group of equations (B) allow to determine already known parameters of the EPS mode ($P_{g1} = 200$ MW, $P_{g2} = 110$ MW, $P_{g3} = 100$ MW, $P_{g4} = 90$ MW). From group of the equations (A) defines the indefinite Lagrange multipliers: $\lambda = 100$, $\nu_1 = -55$, $\nu_3 = 20$, $\mu_5 = -40$.

According to the formula (14) receive values of electricity nodal prices for considered EPS. We have

$$c_n = \begin{pmatrix} c_{n1}\\ c_{n2}\\ c_{n3}\\ c_{n4}\\ c_{n5}\\ c_{n6} \end{pmatrix} = \begin{pmatrix} 100\\ 100\\ 100\\ 100\\ 100\\ 100 \end{pmatrix} + \begin{pmatrix} -55\\ 0\\ 20\\ 0\\ 0\\ 0 \end{pmatrix} + \begin{pmatrix} 0\\ 0\\ -40\\ -40\\ 0\\ -40 \end{pmatrix} = \begin{pmatrix} 45\\ 100\\ 80\\ 60\\ 100\\ 60 \end{pmatrix}.$$

As can be seen from the solution of optimization problem the price in load node 5 is assigned equal to 100 €/MWh. This price is determined by the most expensive generator 2, which was the latter chosen in the market. For the price of load node 5 is assigned nodal load price for the remaining nodes of EPS left part (nodes 1 and 2). For all nodes of the EPS right part (nodes 3,4 and 6) nodal price is assigned equal to the price application of generator 4 with the lowest price, as due to external constraint, it is not fully loaded (has not reached the limit $P_{g4}^{\max} = 200$ MW). The optimization problem final results of competitive market, where predicted electricity demand is satisfied are displayed in fig. 3

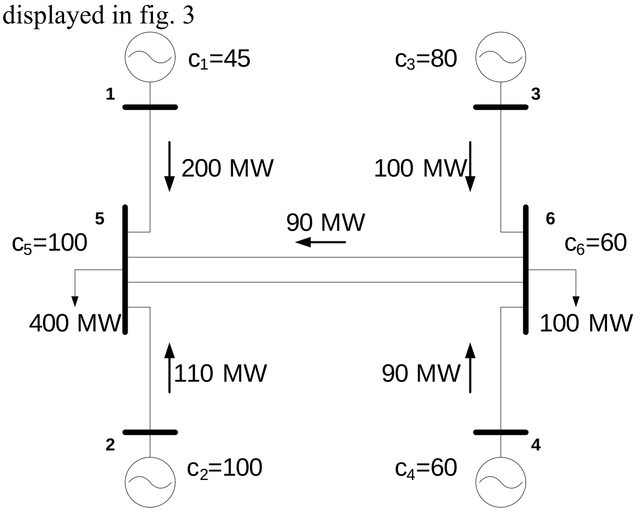

Fig.3. Optimum mode EPS

V. CONCLUSIONS

1. The method of uncertain Lagrange multipliers allow to determine nodal prices of electricity in the EPS load nodes after preliminary determination of the original optimization problem active restrictions of competitive market.

2. Active restrictions revealing of optimization problem using communication linear models between nodal powers and flows on lines can be carried out using the office software of modern computers.

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