A Method for Solving the Time-Periodic Electromagnetic Field Problem in Ferromagnetic Shielding

Iosif Nemoianu*, Florea Ioan Hantila *, Mihai Maricaru*, Dan Rucinschi* and Teodor Leuca**

* Department of Electrical Engineering,
Politehnica University of Bucharest, Faculty of Electrical Engineering,
Spl. Independentei 313, 060042 Bucharest, Romania, E-Mail: hantila@elth.pub.ro

** Academy of Romanian Scientists Technical Sciences Section, University of Oradea
Faculty of Electrical Engineering and Information Technology
Department of Electrical Engineering, Electrical Measurements and Electric Power Use
Str. Universitatii 1, 410087, Oradea, Romania, E-Mail: tleuca@uoradea.ro

***Abstract*—Ferromagnetic shield is an efficient solution for shielding the static and periodic electromagnetic field. Therefore, it is also a shilding solution for an electromagnetic field having a rich contain of harmonics. Using the polarization fixed point method, the nonlinear media is replaced by a linear one having the vacuum permeability and a magnetization that is iteratively corrected by the flux density. For each harmonic of the magnetization, the electromagnetic field may be obtained by solving the sinusoidal steady-state eddy-current equation in the shield. The solution process can be started by retaining a small number of harmonics and, finaly, the acuracy of the solution may be improved with adding some more. The proposed method always yields to stable results, even when the characteristic *B-H* is strongly nonlinear, and has a superior computational efficiency with respect to various time-stepping techniques and to the "harmonic balance method".**

Keywords:* *ferromagnetic shield, eddy-current equation, nonlinear periodic fields, polarization fixed point method, fourier analysis.

I. INTRODUCTION

The analysis of a time-periodic electromagnetic field in nonlinear magnetic media can simply be done by linearizing the ***B-H*** relationship and by correcting iteratively the material permeability [1], but the convergence of the computational process is not always guaranteed. A straightforward stepping-on-in-time transient analysis follows the actual nonlinear relationship $\boldsymbol{B}$ - $\boldsymbol{H}$, but the necessary computation time to reach the periodic steady state may be prohibitive long. The "harmonic balance method" [2] employs a Fourier series expansion of the unknown quantities and yields to large systems of nonlinear algebraic equations whose solution require a huge computational effort. An efficient method to obtain the solution of nonlinear time-periodic electromagnetic field problems is presented in [3], where the magnetic nonlinearity is iteratively treated by the Polarization Fixed Point Method (PFPM)[4].

In this paper, we employ the eddy-current equation for solving the time-periodic electromagnetic field problem in ferromagnetic shielding. Following the PFPM scheme, the nonlinear media is replaced by a linear one having the vacuum permeability and a magnetization that is iteratively corrected by the flux density. Using the Fourier decomposition of the magnetization, each harmonic of the electromagnetic field is computed by solving the complex form of the eddy-current equations, the sources being the imposed current and the magnetization harmonics. Having the eddy-currents, we may obtain the harmonics and the time evolution of the flux density. The time values of the magnetization are corrected by the flux density. The number of linear systems of equations that is to be solved, at each iteration step, is given by the number of harmonics taken into account, which makes this method very efficient. The iterative scheme can be started with a small number of harmonics in order to further increase efficiency.

II. TREATMENT OF THE B-H NONLINEAR RELATIONSHIP

The nonlinear relationship $\boldsymbol{H} = \mathbf{F}(\boldsymbol{B})$ is replaced by

$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M}) \tag{1}$$

where $\boldsymbol{M}$ has a nonlinear dependence of $\boldsymbol{B}$ [4],

$$\boldsymbol{M} = \nu_0 \boldsymbol{B} - \mathbf{F}(\boldsymbol{B}) \equiv \mathbf{G}(\boldsymbol{B}) \tag{2}$$

where $\nu_0 = \dfrac{1}{\mu_0}$ and the function $\mathbf{G}$ is a contraction, i.e.

$$\left\| \mathrm{G}(\boldsymbol{B}_1) - \mathrm{G}(\boldsymbol{B}_2) \right\|_{\mu_0} \le \lambda \left\| \boldsymbol{B}_1 - \boldsymbol{B}_2 \right\|_{\nu_0} \tag{3}$$

The norm is

$$\|U\|_{\nu_0} = \sqrt{\frac{1}{T}\int_0^T \int_\Omega U \cdot (\nu_0 U) d\Omega\, dt} \qquad (4)$$

with T being the period and Ω the space region.

The time-periodic $\boldsymbol{M}$ has a Fourier series expansion of the form

$$\boldsymbol{M}(t) = \sum_{n=1,3,\ldots} \left(\boldsymbol{M}'_n \sin(n\omega t) + \boldsymbol{M}''_n \cos(n\omega t)\right). \qquad (5)$$

For the numerical computation, we retain a finite number N of harmonics, $\boldsymbol{M} \cong \boldsymbol{M}_a \equiv \mathbf{Y}(\boldsymbol{M})$, the approximation $\mathbf{Y}$ being non-expansive, i.e.

$$\left\|\mathrm{Y}(\boldsymbol{M}_1) - \mathrm{Y}(\boldsymbol{M}_2)\right\|_{\mu_0} \le \left\|\boldsymbol{M}_1 - \boldsymbol{M}_2\right\|_{\mu_0}. \qquad (6)$$

For each harmonic n of the magnetization $\boldsymbol{M}$, we use the complex representation

$$\boldsymbol{M}_n = \boldsymbol{M}'_n + j\boldsymbol{M}''_n \qquad (7)$$

and we obtain the sinusoidal steady-state electromagnetic field by solving the eddy-current integral equation of the form

$$(\alpha + jn\omega\beta)\boldsymbol{x} = \boldsymbol{c} \qquad (8)$$

where $\boldsymbol{x}$ is the vector of the current density, $\boldsymbol{c}$ is the known complex vector given by the sources and by $\boldsymbol{M}_n$, and α and β are square matrices that do not depend on the harmonic order. Now, for each harmonic the complex flux density

$$\boldsymbol{B}_n = \boldsymbol{B}'_n + j\boldsymbol{B}''_n \qquad (9)$$

may be computed using the Biot-Savart-Laplace formula. From $\boldsymbol{B}_n$, we obtain the time-domain value of the flux density as

$$\boldsymbol{B}(t) = \sum_{n=1,3,\ldots,2N-1} \left(\boldsymbol{B}'_n \sin(n\omega t) + \boldsymbol{B}''_n \cos(n\omega t)\right) \equiv \mathbf{L}(\boldsymbol{M}_a). \qquad (10)$$

It can be shown that $\mathbf{L}$ is also non-expansive. At each step $k \ge 1$ of the proposed iterative process, we perform the following chain of operations

$$\boldsymbol{B}^k \xrightarrow{\mathbf{G}} \boldsymbol{M}^k \xrightarrow{\mathbf{Y}} \boldsymbol{M}_a^k \xrightarrow{\mathbf{L}} \boldsymbol{B}^{k+1},$$

with $\boldsymbol{B}^1$ arbitrarily chosen. Since the composition of $\mathbf{G}$, $\mathbf{Y}$ and $\mathbf{L}$ is a contraction, the iterative process is always convergent. Instead of systems of equations corresponding to each time step in time-domain methods, using the above method, one has to solve only N linear complex systems, at each iteration. In order to further reduce the amount of computation, we start with a small number N of harmonics (even with $N = 1$). Since the inequality (6) is stronger when the number of harmonics is smaller, the rate of convergence is now higher. When an imposed accuracy is reached, we increase the number of harmonics until the resultant field is accurately determined.

III. EDDY-CURRENT INTEGRAL EQUATION

An advantageous feature of the proposed method allows the construction of an integral equation for the current density to be solved at each iteration [5].

For two-dimensional structures this integral equation can be written in the form

$$\rho J(\boldsymbol{r}) + \frac{\mu_0}{2\pi}\frac{\partial}{\partial t}\int_\Omega J(\boldsymbol{r}')\ln\frac{1}{R}dS'$$
$$= -\frac{\mu_0}{2\pi}\frac{\partial}{\partial t}\int_{\Omega_0} J_0(\boldsymbol{r}')\ln\frac{1}{R}dS'$$
$$-\frac{\mu_0}{2\pi}\frac{\partial}{\partial t}\int_\Omega \boldsymbol{k}\cdot\left(\nabla \times \boldsymbol{M}(\boldsymbol{r}')\right)\ln\frac{1}{R}dS' + C_l \qquad (11)$$

where ρ and J are, respectively, the resistivity and the current density in the conducting regions Ω, J_0 is the given current density in the nonferromagnetic coil regions Ω_0, $\boldsymbol{r}$ and $\boldsymbol{r}'$ are the position vectors of the observation and the source points, respectively, $\boldsymbol{R} = \boldsymbol{r} - \boldsymbol{r}'$, $R = |\boldsymbol{R}|$, and $\boldsymbol{k}$ is the longitudinal unit vector. C_l is a constant for each disjoint conducting region l and is determined by specifying its total current. To illustrate the formulation, we choose only one conducting region Ω with a null total current, when $C = 0$. Ω is divided in I subdomains ω_i and Ω_0 in Q subdomains ω_{0q}. Equation (11) is discretized as

$$\rho_m S_m J_m + \frac{\partial}{\partial t}\sum_{i=1}^{I}\beta_{mi}J_i = -\frac{\partial}{\partial t}\sum_{q=1}^{Q}\beta_{0mq}J_{0q}$$
$$+\frac{\partial}{\partial t}\sum_{i=1}^{I}\gamma_{mi}\cdot\boldsymbol{M}_i, \qquad m = 1,2,\ldots, I \qquad (12)$$

where ρ_m, S_m, and J_m are, respectively, the resistivity, the area, and the average value of the current density of the subdomain ω_m, J_{0q} is the imposed current density in the subdomain ω_{0q}, $\boldsymbol{M}_i$ is the magnetization in ω_i, and

$$\beta_{mi} = \frac{\mu_0}{2\pi} \int_{\omega_m} \int_{\omega_i} \ln \frac{1}{R} dS_i' dS_m$$

$$= \frac{\mu_0}{8\pi} \oint_{\partial\omega_m} \oint_{\partial\omega_i} R^2 \ln R d\boldsymbol{l}_m \cdot d\boldsymbol{l}_i' \quad (13)$$

$$\beta_{0mq} = \frac{\mu_0}{2\pi} \int_{\omega_m} \int_{\omega_{0q}} \ln \frac{1}{R} dS_q' dS_m$$

$$= \frac{\mu_0}{8\pi} \oint_{\partial\omega_m} \oint_{\partial\omega_{0q}} R^2 \ln R d\boldsymbol{l}_m \cdot d\boldsymbol{l}_q' \quad (14)$$

$$\gamma_{mi} = -\frac{\mu_0}{2\pi} \int_{\omega_m} \oint_{\partial\omega_i} \ln \frac{1}{R} d\boldsymbol{l}_i' dS_m$$

$$= \frac{\mu_0}{8\pi} \oint_{\partial\omega_m} \oint_{\partial\omega_i} (2\ln R - 1)(\boldsymbol{R} \cdot \boldsymbol{n}_m) dl_m d\boldsymbol{l}_i' \quad (15)$$

$\partial\omega_i$ is the boundary of the subdomain ω_i and $\boldsymbol{n}_i$ is the outward normal unit vector on $\partial\omega_i$. The system (12) can be written for each harmonic *n* in a matrix form as

$$\begin{pmatrix} \beta & \delta/n \\ -\delta/n & \beta \end{pmatrix} \begin{pmatrix} J'_n \\ J''_n \end{pmatrix} = -\begin{pmatrix} A'_{0n} \\ A''_{0n} \end{pmatrix} + \begin{pmatrix} A'_{Mn} \\ A''_{Mn} \end{pmatrix} \quad (16)$$

where β is the matrix of β_{mi}, δ is a diagonal matrix with the entries $\delta_{mm} = \rho_m S_m / \omega$, $m = 1,2,..., I$, J'_n and J''_n are the column vectors of the real and imaginary parts of the complex current density J_n, A'_{0n}, A''_{0n} and A'_{Mn}, A''_{Mn} are, respectively, the column vectors of the real and imaginary parts of the complex vector potentials integrated over the respective subdomains ω_m, A_{0n} due to the imposed current density and A_{Mn} to the magnetization, i.e.

$$A_{0n} = \sum_{q=1}^{Q} \beta_{0mq} J_{0n,q} \quad (17)$$

$$A_{Mn} = \sum_{i=1}^{I} \gamma_{mi} \cdot \boldsymbol{M}_{n,i} \ . \quad (18)$$

A_{0n} is the same for all iterations, while A_{Mn} is to be corrected at each iteration.

After solving system (16), the complex flux density is obtained from

$$\boldsymbol{B}_n = -\frac{\mu_0}{2\pi} \boldsymbol{k} \times \sum_{i=1}^{I} J_{n,i} \oint_{\partial\omega_i} \ln R d\boldsymbol{l}_i'$$

$$-\frac{\mu_0}{2\pi} \boldsymbol{k} \times \sum_{q=1}^{Q} J_{n,q} \oint_{\partial\omega_{0q}} \ln R d\boldsymbol{l}_q'$$

$$-\frac{\mu_0}{2\pi} \sum_{i=1}^{I} \oint_{\partial\omega_i} \frac{\boldsymbol{R}}{R^2} (\boldsymbol{M}_{n,i} \cdot d\boldsymbol{l}_i') \ . \quad (19)$$

The average value of the complex flux density in the subdomain ω_m is computed as

$$\boldsymbol{B}_{n,m} = -\frac{1}{S_m} \left(\sum_{i=1}^{I} \gamma_{mi} J_{n,i} + \sum_{i=1}^{I} \overline{\overline{\zeta}}_{mi} \boldsymbol{M}_{n,i} \right) + \boldsymbol{B}_{0n,m} \quad (20)$$

where $\boldsymbol{B}_{0n,m}$ is the flux density due to the imposed current density, the same at all the iterations,

$$\boldsymbol{B}_{0n,m} = -\frac{1}{S_m} \sum_{q=1}^{Q} \gamma_{mq} J_{0n,q} \quad (21)$$

and

$$\overline{\overline{\zeta}}_{mi} = \frac{\mu_0}{2\pi} \oint_{\partial\omega_m} \oint_{\partial\omega_i} \ln R (d\boldsymbol{l}_m d\boldsymbol{l}_i') \quad (22)$$

the latter being expressed in terms of the dyads $(d\boldsymbol{l}_m d\boldsymbol{l}_i')$. The numerical approximation of $\boldsymbol{B}_{n,m}$ due to averaging is non-expansive and, thus, preserves the convergence of the iterative process, while the system (16), corresponding to the integral equation (11), could perturb the convergence in the case of large differences in differential magnetic permeability. At any time *t*, the flux density is obtained with (10), the magnetization is corrected with (2) and, then, used to compute the new complex expression in (7), with

$$\boldsymbol{M}_n' = \frac{2}{T} \int_0^T \boldsymbol{M}(t) \sin(n\omega t) dt \ ,$$

$$\boldsymbol{M}_n'' = \frac{2}{T} \int_0^T \boldsymbol{M}(t) \cos(n\omega t) dt \ . \quad (23)$$

IV. ILLUSTRATIVE EXAMPLE

Let us consider the U form shield depicted in Fig. 1, having a 5mm thickness, 300mm wide and 100mm long arms. The ferromagnetic material used has a *B-H* characteristic plotted in Fig. 2. The two parallel conductors carry opposite direction currents, each of 800A-turns, at 50Hz. Field lines for $\omega t = 90^0$ are presented in Fig. 1. Flux densities in the two cases (without the shield B_0 and *B* with the shield) are plotted along a 320 mm wide line placed at 4.5 mm above the shield (Fig. 3). The shielding efficiency is computed using the following relation:

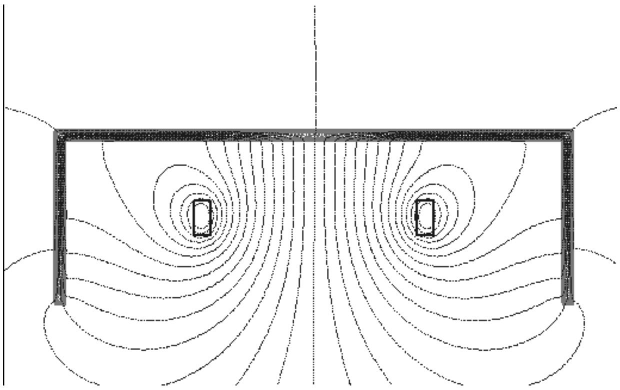

Fig.1. U-form ferromagnetic shield.

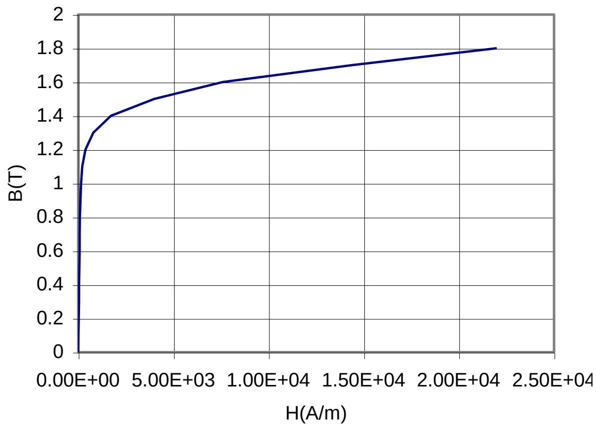

Fig.2. *B*-*H* characteristic.

$$ecr = \sqrt{\frac{\sum_{k=1}^{N} B_k^2}{\sum_{k=1}^{N} B_{0k}^2}} \qquad (24)$$

where N is the number of the selected points on the line placed behind the shield. We obtain ecr = 0,233.

V. REMARKS AND CONCLUSION

Computation was performed by employing a PC with a 2.8 GHz processor and 2GB of RAM. Firstly, we

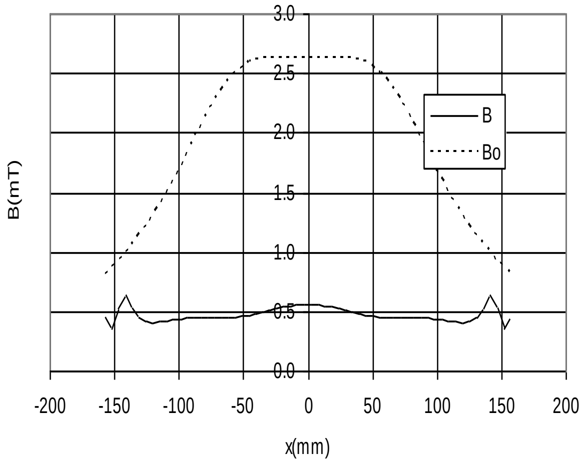

Fig.3. Flux densities behind the shield

consider only the fundamental harmonic and we need 30 iterations for an error of 0.12%. To obtain a higher accuracy, we add the 3-th harmonic. The solution is very close to the first one because the ferromagnetic domain occupies only a small area in the path of magnetic lines.

The same problem was solved by a hybrid FEM-BEM technique [7], and the same solution was obtained.

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