Statistical Images Segmentation

Sorin Curilă*, Mircea Curilă*, Corina Curilă, Diana Curilă *****

* University of Oradea,
str. Armatei Române nr. 5, zip code 410087 Oradea, România, E-Mail: scurila@uoradea.ro

**Onisifor Ghibu College
Aleea Onisifor Ghibu, nr. 3, Oradea, România

***Colegiul tehnic "Mihai Viteazul",
str. Poieniței, nr. 26, Oradea, România

***Abstract* – This paper deals with fuzzy statistical image segmentation. We introduce a new hierarchical Markovian fuzzy hidden field model, which extends to the fuzzy case the classical Pérez and Heitz hard model. Two fuzzy statistical segmentation methods related with the model proposed are defined in this paper and we show via simulations that they are competitive with, in some cases than, the classical Maximum Posterior Mode (MPM) based methods. Furthermore, they are faster, which will should facilitate extensions to more than two hard classes in future work. In addition, the model proposed is applicable to the multiscale segmentation and multiresolution images fusion problems.**

***Keywords:* statistical image, Maximum Posterior Mode, segmentation**

1. INTRODUCTION

This paper deals with fuzzy statistical image segmentation. We adopt the recent Hidden Fuzzy Markov Random Fields (HFMRF) model considered in [12, 13], which simultaneously models the imprecision of membership of pixels to a given class, a fuzzy aspect [3, 10, 14] , and the uncertainty of their belonging to a given class, a probabilistic aspect. Such a mixed approach differs from both a purely fuzzy approach [10], and from purely probabilistic Markov Field Model based approach [2, 6, 7, 9]. Let us briefly specify the interest of fuzzy segmentation in some real situations. Let us consider the problem of segmenting a satellite image into two classes: "houses" and "trees". There may be some pixels with only houses and others with only trees, but there may also be many pixels, as in suburbs, in which houses and trees are simultaneously present. Thus we have two hard classes, say 0 and 1, and a fuzzy class specified by $\varepsilon \in]0,1[$, which can be seen as the proportion of the area of class 1. Now, if we wish to use some statistical method we have to introduce a probability measure p on [0,1]. According to one's intuition, $p[0]$ and $p[1]$ can be strictly positive, but any element of]0,1[can also occur. This is modelled by considering that p is defined with a density h with respect to the measure $\nu = \delta_0 + \delta_1 + \mu$, which includes a "hard" component (Dirac functions δ_0 , δ_1 on $\{0,1\}$), and a "fuzzy" one, which is the Lebesgue measure μ on $]0,1[$. Such modelling was first introduced in local segmentation methods [3, 4] and then generalized to Markovian methods [10, 11]. Fuzzy segmentation methods presented in [10,11] are quite efficient, although the computational burdin can be prohibitive. Thus the aim of the present paper is to propose faster methods. Returning to the classical hard case, it is well known that simulated annealing [7] is time expensive and the Iterated Conditional Mode, ICM [2], which is a fast approximation of MAP, is often used. The problem is that ICM is sensitive to the initialisation and, when poorly initialized, can give poor results. To remedy this, Perez and Heitz proposed a hierarchical structure, which is a set of compatible Markovian fields [11]. Roughly speaking, the solution given with ICM at a given scale serves to initialise ICM at the finest scale. In this work we adapt the Perez and Heitz model to the fuzzy case and show, via simulation, its interest in the fuzzy segmentation. The organization of the paper is as follows: In the next section we shortly recall modelling by Hidden Fuzzy Markov Fields, as proposed in [12, 13]. Section three is devoted to the new hierarchical model we propose and some related segmentation methods are specified in section four. Section five is devoted to the numerical results obtained and section six contains the conclusion.

A. Distribution X

With V a neighbourhood and N the number of pixels, we consider a function $h:[0,1]^N \to R$ of the following form:

$$h(x) = ce^{-U(x)} \quad (2.1)$$

Here U, called "energy", is a sum of functions defined on cliques, a clique being either a singleton or a set of neighbour pixels with respect to V. We consider the stationary case, i.e., that the functions defining U depend only on the shape of cliques and do not depend on their position in the set of
pixels. Thus, if C is a clique of a given shape and $n = Card(C)$, the associated function φ_C is a function from $\Omega_n = [0,1]^n$ into R. Considering the measure

$$\nu = \delta_0 + \delta_1 + \mu \quad (2.2)$$

where δ_0, δ_1 are the Dirac measures on $\{0,1\}$, and μ is the Lebesgue measure on [0,1], we assume that h defined by (2.1) is a density of P_X with respect to $\nu^{\otimes N}$. Thus one can classically show that X is a Markovian field.

B. Distribution of Y conditional to X

We assume that:

(i) The random variables (Y_s) are independent conditionally on X;

(ii) The distribution of each Y_s conditional to X is equal to its distribution conditional to X_s. Distributions of Y conditional to X are then defined by distributions of Y_s conditional to X_s. Denoting by $N(m, \sigma^2)$ the normal distribution of mean m and variance σ^2, we take for the distribution of Y_s conditional to $X_s = x_s \in [0,1]$:

$$N((1-x_s)m_0 + x_s m_1, (1-x_s)\sigma_0^2 + x_s \sigma_1^2) \quad (2.3)$$

Thus the parameters m_0, m_1, σ_0^2, σ_1^2 define all distributions of Y conditional to X. Let ψ_{x_s} be the Gaussian density defined by (2.3). The density ψ of the distribution of (X,Y) with respect to $\nu^{N} \otimes \mu^{N}$ (ν being the measure on [0,1] defined by (2), μ the Lebesgue measure on R and N the number of pixels) is then given by

$$\psi(x, y) = c e^{-U_f(x)} \prod_{s \in S} \psi_{x_s}(y_s) = c e^{-W_y(x)} \quad (2.4)$$

with $W_y(x) = U_f(x) + V_x(y)$ and $V_x(y) = \sum_{s \in S} Log\, \psi_{x_s}(y_s)$.

2. HIDDEN FUZZY MARKOV FIELDS

In this section, we briefly recall the model presented in [12, 13]. We consider two random fields $X = (X_s)_{s \in S}$ and $Y = (Y_s)_{s \in S}$, with each X_s taking its values in $\Omega_f = [0,1]$ and each Y_s taking its values in R. As usual, the probabilistic link between X and Y is modelled by $P_{(X,Y)}$, which is the distribution of (X,Y) and which is defined by the distribution of X and the distributions of Y conditional to X.

C. A posteriori distribution of X

The density of the a posteriori distribution of X (i.e., conditional to $Y = y$) with respect to ν^N is thus given by

$$\psi_y(x) = \frac{k e^{-W_y(x)}}{\int_{[0,1]^N} k e^{-W_y(x)} d\nu^N(x)} \quad (2.5)$$

which can be written as

$$\psi_y(x) = k(y) e^{-W_y(x)} = k(y) e^{-(U_f(x) + V_x(y))} \quad (2.6)$$

As in the case of hard Markovian fields, the Markovian nature of the posterior distribution of X is thus preserved and one can use the Gibbs sampler in order to simulate its realizations.

3. HIERARCHICAL FUZZY MARKOV FIELDS

We assume that $Card(S) = 4^n$ and consider
sequence the classical pyramid sequence in which each "father" has four "so ns", which form, at the lower lever, a block of four elements. So, there are $n+1$ scales : $S_n = S$, S_{n-1} obtained from S_n and having 4^{n-1} elements, ..., S_i having 4^i elements, .., and S_0, which is the top of the pyramid, having $4^0 = 1$ element. Then we consider $n+1$ random fields $X^0, ..., X^n$, with $X^i = (X_s^i)_{s \in S_i}$, such that each variable X_s^i takes its values in [0,1]. Thus the realizations of X^i can be seen as particular realizations of X^{i+1} in the following sense :

$$[X_s^i = x_s^i] \Leftrightarrow [\textit{for all t sons of s } X_t^{i+1} = x_t^{i+1}] \quad (3.1)$$

Let us assume that we dispose of some iterative segmentation method $\hat{s}$, like ICM, and the drawback is that $\hat{s}$ is sensitive to the initialization. Roughly speaking, the idea of Pérez and Heitz is then to use the result obtained with $\hat{s}$ at the scale I in order to initialize $\hat{s}$, using (3.1), at the scale $i+1$. Subject to some compatibility of Markovian structures on different scales, the method they proposed gives good results in the classical hard case. We present below an adaptation of the hierarchical hidden Markovian structure of Perez and Heitz to the fuzzy model. We place ourselves at the base of the pyramid and consider the markovianity relative to the four nearest neighbours. The energy is then defined with functions φ_C, where C is either a singleton or a set $\{s,t\}$ of neighbours. They will be assumed null on singletons, and

$$\varphi_{\{s,t\}}(x_s, x_t) = \begin{cases} -\alpha & \text{if } x_s = x_t \\ \alpha & \text{if } x_s \neq x_t \end{cases} \quad (3.2)$$

for $(x_s, x_t) \in \{0,1\}^2$, $\varphi_{\{s,t\}}(x_s, x_t) = -\beta(1 - 2|x_s - x_t|)$ (3.3)
for $(x_s, x_t) \in [0,1]^2 - \{0,1\}^2$

The fuzzy class field is then noise corrupted as specified in section 2.2 above. Now, let us consider a scale i. We will define an energy φ^i "compatible" with the energy defined by (3.2), (3.3). Each pixel s_i of S_i contains $p_i = 2^{i+1}(2^i - 1)$ binary cliques of the basic pixels. According to (3.2), (3.3) we thus have for singleton cliques

$\varphi_{I\{s_i\}}(x_{s_i}) = -p_i\,\alpha \; if\, x_{s_i} \in \{0,1\} \; -p_i\,\beta \; if\, x_{s_i} \in]0,1[$ (3.4)

On the other hand, there are $q_i = 2i$ basic binary cliques touching s_i and t_i for a given binary clique $\{s_i, t_i\}$ of S_i. Associated with binary cliques, the functions φ_i are

$\varphi_{I\{s_i,t_i\}}(x_{s_i}, x_{t_i}) = -q_i\,\alpha \; if\, x_{s_i} = x_{t_i}\, q_i\,\alpha \; if\, x_{s_i} \neq x_{t_i}$ (3.5)

for $(x_{s_i}, x_{t_i}) \in \{0,1\}$,

$$\varphi_{I\{s_i,t_i\}}(x_{s_i}, x_{t_i}) = -q_i\,\beta(1-2\,x_{s_i} - x_{t_i}) \quad (3.6)$$

for $(x_{s_i}, x_{t_i}) \in [0,1]^2 - \{0,1\}^2$ and the noise inferred at the scale i is given by $f_{x\,si}$

$$(y_{s_i}) = f_{x_s}(y_s)\; {}_{s\in s_i}\prod \quad (3.7)$$

Finally, (3.4)-(3.7) define a set, indexed to scales, of hidden Markov fuzzy fields.

4. CLASSICAL AND HIERARCHICAL FUZZY MARKOV FIELDS BASED SEGMENTATIONS

We recall in this section two classical Fuzzy Hidden Markov Field based segmentation method and two new Hierarchical Fuzzy Hidden Markov Field based ones.

A. Fuzzy MPM methods

According to the model specified in the section 2, simulations of the fuzzy field X conditional to Y are possible, and thus one can estimate the posterior marginal of X. Once the distribution of X_s conditional to $Y = y$, given by a density h_s with respect to the measure $\nu = \delta_0 + \delta_1 + \mu$, is known the following two methods can be considered:

1) The Fuzzy MPM1 (FMPM1) method is defined with

$\hat{x} = FMPM1(y)\; [\;] \Leftrightarrow \forall s \in S\; h_s(\hat{x}_s) = \sup_{t\in[0,1]} h_s(t)$ (4.1)

2) The Fuzzy MPM2 (FMPM2) is defined with $\hat{x} = FMPM2(y)\; [\;] \Leftrightarrow \forall s \in S,\; \hat{x}_s = 0 \; if \; h_s(0) \geq \sup(h_s(1), 1 - h_s(0) - h_s(1))\; 1 \; if\; h_s(1) \geq \sup(h_s(0), 1 - h_s(0) - h_s(1))\; \mathrm{argmax}_{t\in]0,1[} h_s(t) \; otherwise$ (4.2) FMPM1 and FMPM2 give satisfying results, although the visual effects of the segmented images by the both methods can be different.

B. Hierarchical Fuzzy ICM methods First, let us specify how Fuzzy ICM1 (FICM1) and Fuzzy ICM2 (FICM2) run. According to section 2, the distribution of X conditional to $Y = y$ is a Markov distribution and thus the distribution of each X_s conditional on $(X_{t1}, X_{t2}, X_{t3}, X_{t4})$, where t_1, t_2, t_3, t_4 are neighbours of s, is computable. In FICM1 we scan the set of pixels and, at each pixel s, we replace the current fuzzy value by that which maximises the density, with respect to $\nu = \delta_0 + \delta_1 + \mu$, of the distribution of X_s conditional to $(X_{t1}, X_{t2}, X_{t3}, X_{t4}) = (x_{t1}, x_{t2}, x_{t3}, x_{t4})$, where $x_{t1}, x_{t2}, x_{t3}, x_{t4}$ are the current values of neighbours of s. Thus FICM1 runs like the classical hard ICM, with the difference that one maximises a density with respect to $\nu = \delta_0 + \delta_1 + \mu$ instead of maximizing a finite probability. In the FICM2 we still scan the set of pixels and, at each pixel s, we consider the density, with respect to $\nu = \delta_0 + \delta_1 + \mu$, of the distribution of X_s conditional to $(X_{t1}, X_{t2}, X_{t3}, X_{t4}) = (x_{t1}, x_{t2}, x_{t3}, x_{t4}) = x_{V_s}$. Denoting this density by $h_{V_s s}$, the new fuzzy value of s is chosen using (4.2), in which h_s is replaced by $h_{V_s s}$. The Hierarchical FICM1 and Hierarchical FICM2 (HFICM1 and HFICM2 respectively) methods are then obtained from FICM1 and FICM2 using the hierarchical structure in the following way :

(i) use FICM1 (respectively, FICM2) at the scale r, which is the top of the pyramid ($r = n$), or near the top.

(ii) initialize FICM1 (respectively, FICM2) at the scale $i-1$ with the segmentation found at the scale i.

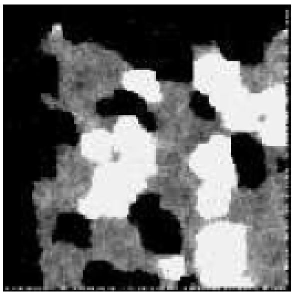
(iii) obtain the final segmentation at the scale $i = 0$ (the base of the pyramid)

5. SIMULATION RESULTS

We present in this section three series of results concerning three fuzzy images : one realization of a Fuzzy Markov Random Field, and two hand written ones. Each of them is corrupted by a Gaussian noise and then segmented with the four methods MPM1, MPM2, HFICM1, and HFICM2. The performance of each is evaluated visually and with the error rate τ, which is defined by

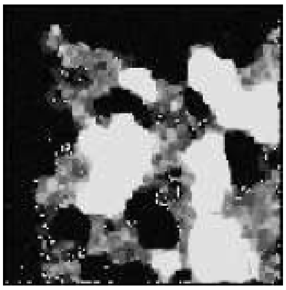
$$\tau = 1\; N\; \hat{x}_s - x_s\; {}_{s\in S}\sum \quad (5.1)$$

The results are presented in Figures 1 and 2. The most striking impression is that the hierarchical methods are visually better behaved than the MPM methods. This is true in the fuzzy Markov Field realization case as well as in the hand written fuzzy image case. Other results presented in [1] confirm these impressions. On the other hand, the visual aspects of different methods can be quite different. This can lead to the conclusion that each method can be useful in some particular situation. For instance, HFICM 1 seems to better restore the hard classes, while HFICM2 better renders the fuzzy classes. Thus, when we are mainly interested in detecting spots with hard classes, HFICM1 is better suited, but HFICM2 should be chosen when we are interested in fuzzy spots.



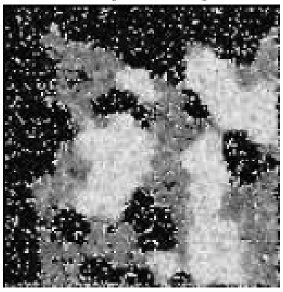
Fuzzy Markov Field

Noisy field. Noise variances equal to 1, means 0 and 4.

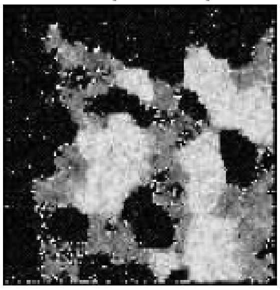


Segmentation with HFICM1, τ= 11,6%

Segmentation with HFICM2, τ= 12,2%



Segmentation with MPM1, τ= 16,9%



Segmentation with MPM2, τ= 14,4%

Figure 1

Fuzzy Markov Field realization, its noisy version, and segmentations with MPM1, MPM2, HFICM1, and HFICM2. τ designates the error ration defined with (5.1).

6. REFERENCES

[1] L. Amoura, Modèle markovien pyramidal flou et segmentation statistique d'images, Ph. D Dissertation, Université Pierre et Marie Curie, Paris, February, 3, 1998.

[2] J. Besag, On the statistical analysis of dirty pictures, *Journal of the Royal Statistical Society*, Series B, 48, 1986, pp. 259-302.

[3] J. C. Bezdek, Pattern Recognition and Fuzzy Objective Function Algorithm. Plenum Press, New-York, 1981.

[4] H. Caillol, A. Hillon, and W. Pieczynski, Fuzzy random fields and unsupervised image segmentation, *IEEE Transactions on Geoscience and Remote Sensing*, Vol. 31, No 4, 1993, pp. 801-810.

[5] H. Caillol, W. Pieczynski, and A. Hillon, Estimation of Fuzzy Gaussian Mixture and Unsupervised Statistical Image Segmentation, *IEEE Transactions on Image Processing*, Vol. 6, No. 3, 1997.

[6] R. Chellapa and A. Jain (ed.), Markov Random Fields, *Academic Press*, 1993.

[7] S. Geman and D. Geman, Stochastic relaxation, Gibbs distributions and the Bayesian restoration of images, *IEEE Transactions on PAMI*, Vol. 6, No. 6, 1984, pp.721-741.

[8] J. T. Kent and K. V. Mardia, Spatial Classification Using fuzzy Membership Models, *IEEE Transaction on PAMI*,Vol. 10, No. 5, 1988, pp. 659-671.

[9] J. L. Marroquin, S. Mittle, and T. Poggio, Probabiliste solution of ill-posed problems in computational vision, *Journal of the American Statistical Association*, Vol.82, 1987, pp.76- 89.

[10] W. Pedrycz, Fuzzy sets in pattern recognition: methodology and methods, *Pattern Recognition,* Vol. 23, No. 1/2, 1990, pp. 121-146.

[11] P. Pérez and F. Heitz, Une approche multiéchelle à l'analyse d'images par champs markoviens, *Traitement du Signal*, Vol. 9, No. 6, pp. 459-472, 1992.

[12] W. Pieczynski and J. M. Cahen, Champs de Markov cachés flous et segmentation d'images, *Revue de Statistique Appliquée*, Vol. 42, No 2, 1994, pp. 13-31.