Scattering by an Almost Circular Cylinder With Smooth Surface

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I. INTRODUCTION

The problem of finding the scattered and diffracted fields of a plane wave by an infinite cylinder with arbitrary cross section presents considerable difficulty. We discuss a boundary perturbation method in which we conformally map the given arbitrary surface into one which is more easily treatened. Through conformal mapping the geometrical characterisitcs of the scattering surface are reflected in the behaviour of the mapping fuction on the transformed boundary. The mapping leads to an integral equation which we solve for the particular problems of scattering both by an almost circular cylinder with smooth surface and by discontinuities in curvature.

II. INTEGRAL EQUATION

Consider a plane wave incident on a boundary on which the total field vanishes. The origin of the coordinate system is taken at the geometrical centre of the object and the pozitive real axis along the direction of propagation of the incident wave. Let $U(\rho)$ be the scattered field.

First the domain external to the given boundary, for which the Greens function satisfying the Dirichtel boundary condition is known. For a closed boundary the mapping function is given as:

$$w = F(z) = y + O(1/z) \quad \text{as} \quad |z| \to \infty \tag{1}$$

We set $\quad u(z) = U(F(z)) \qquad (2)$

and $u(z)$ will satisfy a transfomed wave equation and boundary conditions whereas the radiation condition remains invariant. Through application of Greens theorem, u can be reduced to the solution of the following integral equation:

$$u(\bar{r}_1) = \int_{C_z} \frac{\partial G}{\partial n}(\bar{r}_1, \bar{r}) e^{ik \operatorname{Re} F(z)} dl + \int_{D_z}\int G(\bar{r}_1, \bar{r}) \left(\left| \frac{dw}{dz} \right|^2 - 1 \right) u(\bar{r}) k^2 dx dy \tag{3}$$

The transformed domain and boundary are designated as D_z and C_z. The first term can be interpreted physically as field radiated from C_z, due to the source given from the transformed boundary value $e^{ik \operatorname{Re} F(z)}$. The second term represents the effect of the inhomogenity $\left|\frac{dw}{dz}\right|$ in the medium on the propagation of the radiated scattered fields. The inhommogenity $\left|\frac{dw}{dz}\right|$ in the medium keeps both electrical lenght and amplitude of the ray and effective curvature (the curvature of the boundary minus the ray) invariant under conformal mapping.

The solution of the equation (3) can be obtained by the method of successive approximations, provided that both original and transformed boundaries have continuous tangents and that the distance between the two curves is sufficiently small compared to the wavelength. Under these conditions the norm of the kernel is small compared to unity. Once we obtain the solution *u(z)* of the transformed boundary value problem, the far field expression of *U(z)* can be easily obtained since the mapping function becomes an identity as $|z| \to \infty$.

Let us, consider a scattering body with periodic corrugation, such that boundary is given by the polar equation

$$\rho = R + a \cos m\varphi + b \sin m\varphi \qquad (4)$$

If k $\sqrt{a^2+b^2}$ <1, then by mapping the given boundary into a cicular one with radius R the scattered field is obtained by the integral equation method described .

The mappinf of a domain exterior to an almost circular curve to a domain exterior to a circle is given next.

Let the almost circular boundary C_w be, given by:

$$\rho(\theta) = R + r_p(\theta) \qquad (5)$$

where we assume

$$|r_p| \le \delta \le 1$$

If r_p has the Fourier series representation

$$r_p(\theta) = \sum_{n'1} (a_n \cos n\theta + b_n \sin n\theta) \qquad (6)$$

then the desired mapping is given by the equation

$$w=F(z)=z+\sum_{n'1} \frac{a_n + ib_n}{(z/R)^{n-1}} + O(\delta^2) \qquad (7)$$

On the transformed boundary |z| = R,

$$\mathrm{Re}F(z) \approx R\cos\theta + \left[r_p(\theta) - \bar{r}_p(\theta)\sin\theta \right] \qquad (8)$$

Where

$$\bar{r}_p(\theta) = \frac{1}{2\pi} PV \int_0^{2\pi} r_p(t) \cos\frac{t-\theta}{2} dt \qquad (9)$$

We insert equation 7) and (8) into equation (3) and solve the integral equation by the method of succesive approximation for small $k\delta = \sqrt{a^2+b^2}$.

The scattered field is given as

$$U(\rho_1) = U_0(\rho_1) + U_p(\rho_1) \qquad (10)$$

Where $U_0(\rho_1)$ is the field scattered by the circular cylinder with the radius R and

$$U_p(\rho_1) \approx \frac{1}{\pi} \sqrt{\left(\frac{2}{\pi k \rho_1}\right)} e^{ik\rho_1 + i\left(\frac{\pi}{4}\right)} \sum_{n=-\infty}^{\infty} \frac{e^{in\varphi_1}}{H(kR)} \times$$
$$\times \left\{ i^m \frac{(a+ib)}{R(H(kR))} + i^{-m} \frac{(a-ib)}{R(H(kR))} \right\} \qquad (11)$$

is the perturbed field.

From the study of equation (3), the conditions for validity of the solution are now clearly that not only k $\sqrt{a^2+b^2}$ but also the norm of the kernel of (3) should be much less than unity.

We consider the effect of discontinuity in the curvature or in a derivative of the curvature of the scattering surface. We will refer to such discontinuities as edges. We suppose the scattering surface to have a finite number of edges, and we map the given boundary into a curve C_z which is free of edges. Then from the integral equation (3) we proceed to fiind the solution.

Since we wish to make use of the Neumann series solution we eill consider how to achieve mapping which will assure convergence. In priciple we can choose C_z to be identical with the original boundary except in a small neighborhood we perturb the scattering surface so that the edge is smoothed and the kδ remains as small as desired. This perturbation may introduce new edges but these will be of higher order than the original edge. We show below that the effect of discontinuity in the *n*th derivate of the tangent to the curve is a term in the scattered field order $k^{-n}(k\rho_1)^{1/2}$. Hence, the new edge of higher order can be neglected compared with the original edge for high frequency scattering.

Because of the difficulty in finding the local mapping fuction near an edge, we turn to a curve which is near circular. This is not a limitation on the method and we purpose of isolating the effect of an edge this approach is adequate.

As a preliminary to determining the effect of an edge , we turn to a curve which is near circular. This is not a limitation on the method and for the purpose of isolating the effect of an edge this approach is adequate. As preliminary to determining the effect of an edge on the scattering from a near circular curve we need to determine the form of the high frequency approximation to the Green function for a circle.

$$\frac{\partial G(\bar{r}_1,\bar{r})}{\partial r} \cong \sqrt{\left(\frac{2}{\pi k r_1}\right)} e^{ikr_1 - i\pi(\pi/4)} \frac{ik}{4M}$$

$$\sum_{n=0}^{\infty} \left\{ \begin{array}{l} f(\xi_n)\exp\left[ikR\left(\theta_1+\theta-\frac{\pi}{2}+2n\pi\right)\right]+ \\ +f(\xi'_n)\exp\left[ikR\left(\theta_1-\theta-\frac{3\pi}{2}+2n\pi\right)\right] \end{array} \right\} \quad (12)$$

for kR>1, where $M = \left(\frac{kR}{2}\right)^{1/3}$

$$\xi_n = (\theta_1+\theta-\frac{\pi}{2}+2n\pi)M$$

$$\xi_n = (\theta_1-\theta-\frac{3\pi}{2}+2n\pi)M$$

and $f(\xi_n)$ is the Fock function given by:

$$f(\xi) = \frac{1}{\sqrt{\pi}} \int_{\Gamma} \frac{e^{i\xi t}}{w_1(t)} \quad (13)$$

where $w_1(t)$ is the Airy function defined by the equation:

$$w_1(t) = \sqrt{\pi}[Bi(t)+iAi(t)]$$

The Fock function $f(\xi)$ are the surface fields induce don the circular cylinder by the incident wave.

III.REFERENCES

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