A Method of Accelerated Life Testing for Multidimensional Process

Gabriela Tonț and Dan George Tonț
Department of Electrical Engineering, Electrical Measurements and Electric Power Use
University of Oradea, Faculty of Electrical Engineering and Information Technology
Postal address, Universitatii st., no. 1, zip code 410087, Oradea, România, dtont@uoradea.ro

Abstract – More complex processes and products must be developed in shorter time and reliability must be delivered in the first. In order to efficiently obtain performance data, yields reasonable estimates of the products life or performance under normal operating condition, the paper presents a method of reliability test for a multivariable process, generating the density function of the reliability by means of Bayes procedure.

Keywords: Reliability test, failure, random variable, confidence interval.

I. INTRODUCTION

Given the strong drive to develop an accurate accelerated test model, the stress levels are calculated using operating data from a former product near or overlap the normal operating range. The planning and execution of life tests are based on the product requirements concerning reliability and the associated confidence level. Without these specifications tests cannot be performed, at least from the statistical point of view. Specifically, analysis relies on life and stress data or times-to-failure data at a specific stress level. The accuracy of any prediction is directly proportional to the quality of and accuracy of the supplied data.

The classical theory to determine sampling plans yields a large sample-size necessary to demonstrate the product reliability (1). Above all, the sample-size increases tremendously, if failures have to be taken into account. To use all information about the product given through the development process the application of Bayes procedure (2) is recommended. The reliability demonstration test can be planned optimally regarding sample-size and test duration, if information from product development is utilized. Information about the product lifetime and reliability is often available in early stages from fatigue damage calculations, preceding tests or the analysis of warranty data of a former product which can be treated similar with regard to its failure behavior. To consider such information in the planning of subsequent reliability tests, it is necessary to transform the knowledge into a prior distribution of the random variable. In this case the random variable corresponds to the product reliability at the specified lifetime in the field. Afterwards, the posterior distribution of the reliability is generated with the actual sample distribution by means of Bayes procedure (2). Here, the problems occur on the one hand in the definition of the prior distribution and on the other hand in the different sources of information that are used within the planning of the reliability demonstration test.

II. RELIABILITY AS RANDOM VARIABLE

The reliability is a random variable with any possible value in the interval [0,1]. In (1) the probability density function of the failure probability at a defined time point is given by a beta distribution. Hence, the probability density function of the reliability can be written as:

$$f(R)=\frac{1}{\beta(A,B)}R^{A-1}(1-R)^{B-1} \quad (1)$$

The parameters of the beta distribution have to be chosen as follows:

$$A=n-i+1 \text{ and } B=i \quad (2)$$

where n is the sample-size and i is the rank.

A Weibull distribution in the Weibull net is shown in Figure 1.The straight line corresponds to the median value determined from the failure data. Supplementary, the 90%-confidence interval is shown. The confidence interval of the Weibull distribution depends on the given data, i.e. the sample-size. The confidence interval can be derived from the beta probability density function given for different failure times. In Figure 1 the beta probability density function's are shown for the 5th (rank i = 5), 10th (rank i = 10), 15th (rank i = 15), 20th (rank i = 20), and 25th (rank i = 25) failure time. The shapes of the beta probability density function's depend on the rank and the failure time respectively.

If a specification is given for the product reliability, then the requirement corresponds to the minimum value of the reliability R_{min}. In this context, the reliability is associated to a fixed time point, such as the specified lifetime ts of the product. It must be avoided with a certain confidence to fall below $R_{min}(t_s)$. The confidence level is theprobability that the actual reliability R achieves at least the required value, i.e. C = P(R > R_{min}). Generally, the confidence level is a

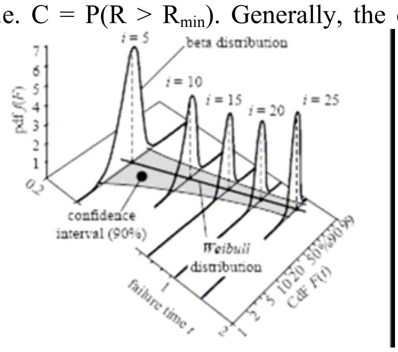

Figure 1: Weibull distribution with beta with probability density function for different ranks (T = 1, b = 1.5, n = 30)

one-sided confidence interval, and can be determined as follows:

$$C = \int_{R=R_{min}}^{1} f(R)dR \quad (3)$$

The classical procedure to determine the confidence level is based on the binomial distribution. The confidence level can be calculated by (1):

$$C = 1 - \sum_{i=0}^{n} \binom{n}{i} R_{min}(t)^{n-1} (1 - R_{min}(t))^{x} \quad (4)$$

where x failures are observed out of n units.

The reliability $R_{min}(t)$ corresponds to the minimum value that has to be achieved by the product at time t.

To determine sampling plans yields a large sample-size necessary to demonstrate the product reliability. Test efforts can be reduced if information about the product given through the development process is considered. The information given before the planning of a reliability test has to be available as a prior probability density function f(R) of the reliability. Then, the posterior probability density function of the reliability is generated with the actual sample distribution by means of Bayes theorem (2, 3):

$$f(R|E) = \frac{P(E|R) f(R)}{\int_0^1 P(E|R) f(R) dR} \quad (5)$$

where P(E/R) is the conditional probability given by a binomial distribution:

$$P(E|R) = \binom{n}{x} R(t)^{n-x} (1 - R(t))^{x} \quad (6)$$

If the prior probability density function of R(t) is assumed to have a uniform distribution in the interval [0,1], the Bayes theorem yields the confidence level as follows (5):

$$C = 1 - \sum_{i=0}^{n} \binom{n+1}{i} R_{min}(t)^{n+1-i} (1 - R_{min}(t))^{x} \quad (7)$$

The uniform prior probability density function can be used if no other prior information is available. If information about the product is given prior to the test, the reliability at the specified product lifetime should be quantified as a beta probability density function given with Eq. (1).

III. DETERMINATION OF PRIOR PROBABILITY DENSITY FUNCTION

Proceeding on the assumption that previous knowledge concerning reliability is available, it is now necessary to define this knowledge by means of a distribution function. Subsequently, it will be suggested how to generate a prior probability density function from preceding tests, the analysis of warranty data of a former product or fatigue damage calculations.

It is considered that prior information is available from preceding tests where no or few failures occurred. The test results serve for the definition of a beta prior probability density function where the parameters are given with Eq. (2). If the test is performed without failure, it is assumed that the rank is i = 1. This means theoretically that one test item is about to fail at the time point of the test end. This corresponds to the worst case. The actual first failure can occur substantially later. If one failure occurred before the test end is reached, it is assumed that the second failure will be exactly at the time point where the test end is achieved. Generally, the rank is given with i = x + 1,where x is the number of failures.The remarks so far are limited to preliminary tests, where the test time tt is equivalent to the specified product lifetime t_s. As a result, the prior probability density function is valid for the interesting time point, namely the specified product lifetime. To reduce sample-size and test duration components are exposed to much higher stresses during the test than within their normal use conditions.Additionally, it has to be taken into account that the test time may vary from the required product lifetime. For both cases it is necessary to transform the given information into the specified product lifetime in order to assure that the beta prior probability density function corresponds to the specified lifetime, Figure 2.Therefore, an acceleration factor $\chi = t'/t_t$ and a lifetime ratio $L_r = t_t/t_s$ has to be considered(6). In this context, the failure distribution of the product has to be estimated. Subsequently, the Weibull distribution is used for its description (1, 4).

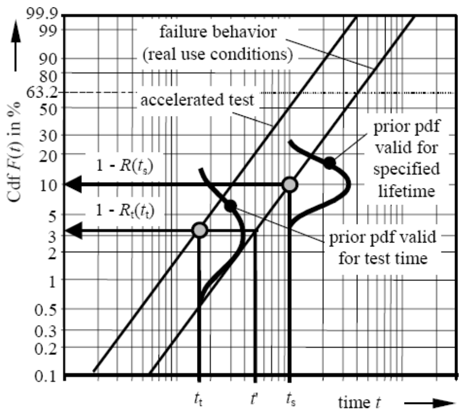

Figure 2: Description of the prior probability density function given from an accelerated test up to a test time that is not equivalent to the specified lifetime and the prior probability density function valid for real use conditions at the specified lifetime

The required reliability in the field $R(t_s)$ depends on the reliability at a given test time $R_t(t_t)$, the acceleration factor the lifetime-ratio L_r, and the shape parameter b of the Weibull distribution (5):

$$R(t_s) = R_t(t_t)^{\frac{1}{(\chi l)^a}} \quad (8)$$

The general definition of the parameters of the beta prior probability density function, given with Eq. (2), is still valid. However, it has to be considered that the rank i changes due to the given in Eq. (8). The approximation for the median rank of the beta distribution $F = (i-3)/(n-0.4)$ and Eq. (2) yield the parameters of the beta prior probability density function:

$$A_0 = n - \left(1 - R_t(t_t)^{\frac{1}{(\chi l,\gamma)^a}}\right)(n+0.4) + 0.7 \text{ and}$$

$$B_0 = n - \left(1 - R_t(t_t)^{\frac{1}{(\chi l,\gamma)^a}}\right)(n+0.4) + 0.3 \quad (9)$$

The reliability $R_t(t_t)$ in Eq. (9) corresponds to the median value that results from the analysis of the preliminary test with a confidence level of C = 0.5. For a preliminary test without failures the parameters of the beta prior probability density function are defined as:

$$A_0 = n - \left(1 - 0.5^{\frac{1}{n(\chi l,\gamma)^a}}\right)(n+0.4) + 0.7 \text{ and}$$

$$B_0 = \left(1 - 0.5^{\frac{1}{n(\chi l,\gamma)^a}}\right)(n+0.4) + 0.3 \quad (10)$$

A reduction of test efforts may be possible if information about the failure behavior of a former product in field operation is available. Such information can be generated by a statistical analysis of warranty data (7). The failure probability at the specified product lifetime is given with:

$$F(t_s) = 1 - e^{-\left(\frac{t_0 - \chi_0}{T - t_0}\right)^b} \quad (11)$$

assuming Weibull distributed failure behavior. Eq. (11) and the approximation for the median rank yield a dependency of the rank on the parameters of the Weibull distribution, the required product lifetime and the sample-size. Then, the parameters of the beta prior probability density function can be obtained by:

$$A_0 = n - (n+0.4)\left(1 - e^{-\left(\frac{t_x - t_0}{T - t_0}\right)^b}\right) + 0.7 \text{ and}$$

$$B_0 = (n+0.4)\left(1 - e^{-\left(\frac{t_x - t_0}{T - t_0}\right)^b}\right) + 0.3 \quad (12)$$

A fatigue damage calculation yields the product lifetime for a certain value of the reliability (1). The desired product lifetime in the field may differ. Therefore, the prior distribution must be transformed into the relevant time point, namely the specified product lifetime. On the assumption that the failure behavior is described by means of a Weibull distribution, the reliability at the desired lifetime can be derived from the results of the fatigue damage calculation as follows:

$$R_{cal}(t_s) = \left(1 - F(t_{cal})\right)^{\left(\frac{t_n}{t_{cal}}\right)^b} \quad (13)$$

In Eq. (13) t_{cal} corresponds to the calculated lifetime, F(tcal) to the failure probability given from the Wöhler-curve (1) and t_s to the required product lifetime. The shape parameter b has to be estimated. Moreover, it is necessary to quantify the reliability determined with Eq. (13) by means of a distribution. Subsequently, a piecewise function is used as prior distribution (Figure 3) which is mathematically written as:

$$f(R) = \begin{cases} \dfrac{R(t_s)}{R_{cal}(t_s)} & 0 \le R < R_{cal}(t_s) \\ \dfrac{1 - R(t_s)}{1 - R_{cal}(t_s)} & R_{cal}(t_s) \le R \le 1 \end{cases} \quad (14)$$

The confidence level achieved with the calculation is suggested to be chosen as the confidence level which results from the uniform distribution (2) and the required reliability $R(t_s)$. This approach only makes sense if the reliability obtained from the calculation is higher than the required reliability. Such an acceptance seems to be legitimate, since safety factors are considered in calculations. Calculation results should be handled with care due to environmental influences which often cannot be considered by the calculation but which the product is exposed to in the field. Therefore, the confidence of the calculation is estimated with a relatively low level compared to other suggestions where the confidence is estimated with 50% .

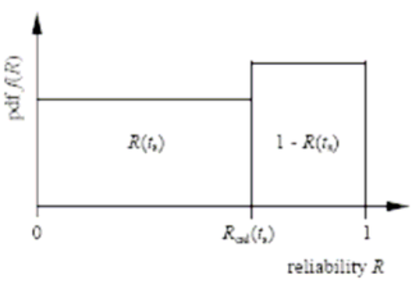

Figure 3: Prior distribution given from calculation

IV. CONSIDERATION OF UNCERTAINTIES IN THE USE OF PRIOR INFORMATION

When prior information is used there is always the uncertainty to what extent the information about the reliability is valid for the actual product conditions. If it can be assumed that the prior conditions and the actual conditions are identical, the prior information about the reliability can be totally transferred for the planning of reliability demonstration tests. As consequence no subsequent reliability demonstration tests would be necessary if the required reliability has already been reached in the past

It is obvious that the information of a former product or preceding tests may not be totally transferable to the actual product concerning reliability. Due to differences in test conditions or environment and function the total use of prior information is not recommended. Otherwise there exists the risk of failing to meet the product requirements. Therefore, it is advisable to consider an uncertainty in the information that is given prior to a test. In this context, it will be assumed that the prior estimation of the reliability is too optimistic. To minimize the risk of missing the product requirements, the so-called "decreasefactor" $\delta$is introduced which artificially reduces the quality level of prior reliability information. The consideration of the decrease-factor yields a modified prior probability density function given as follows:

$$f(R)=\frac{1}{\beta(\delta A_0,\,\delta(B_0-1)+1)}R^{\delta A_0-1}(1-R)^{\delta(B_0-1)} \quad (15)$$

The modified prior probability density function in Eq. (15) is a beta distribution where the modified beta parameters depend on the parameters obtained from prior knowledge and the decrease factor. The parameter A_0 that is determined from prior product information can be interpreted as the number of successes out of a trial. The higher the number of successes gets, the higher becomes the confidence level. To artificially reduce this prior confidence the parameter $A_0$is multiplied with the decrease factor chosen between 0 and 1, and yields the new parameter for the modified prior probability density function with δA_0. Due to the dependence of the beta parameters $A_0$and $B_0$the second modified beta parameter becomes $\delta(B_0-1)+1$. The posterior probability density function is then determined by Bayes theorem with Eq. (5), considering the modified prior probability density function from Eq. (15) and the actual sample distribution from Eq. (6):

$$f(R|E)=\frac{1}{\beta(\delta A_0+n-x,\,\delta(B_0-1)+1+x)}R^{\delta A_0+n-x-1}(1-R)^{\delta(B_0-1)+x} \quad (16)$$

The confidence level can be obtained by integrating Eq.(16):

$$C=\int_{R=R(t_x)}^{1}\frac{1}{\beta(\delta A_0+n-x,\,\delta(B_0-1)+1+x)}R^{\delta A_0+n-x-1}(1-R)^{\delta(B_0-1)+x}\,dR \quad (17)$$

For a decrease-factor $\delta=0$ the information given prior to the test is not used at all. A decrease-factor $\delta=1$ yields the total usage of the prior information. The introduction of the decrease-factor enables the use of prior information but reduces the risk of overestimating the prior probability density function by decreasing the confidence that is actually given from prior knowledge.

Subsequently, a procedure for the estimation of the decrease- factor is suggested for a case where information about the failure behavior of a former product is utilized. The procedure is based on a pragmatic approach where the FMEA (Failure Mode, Effect Analysis) offers a useful foundation on condition that a FMEA is available for both the former product and the actual product (9). The FMEA yields the system structure and the top functions. Failures modes, failure causes and failure effects are derived for every top function. The top functions are of interest for the determination of the decrease-factor. A top function can have several failures modes. A failure mode can have several failure causes. For every failure mode the most risky failure cause is relevant. If the sum of all risk priority numbers of the most risky failure causes determined for a top function of the actual product is higher than the sum determined for the former product, the prior information should be rejected. The information should be considered if the sum of the actual product is equivalent or smaller than the sum of the former product. This yields an indicator as follows:

$$I(F_f)=\frac{\sum_{i=1}^{m}RPN_{\max i}}{\sum_{i=1}^{m_0}RPN_{\max 0}} \quad (18)$$

The portability regarding one top function may than be determined as follows:

$$I(F_f)>1\rightarrow p(F_f)=0 \quad \text{or} \quad I(F_f)\leq 1\rightarrow p(F_f)=1 \quad (19)$$

A vector can be introduced if all top functions of the actual product are considered:

$$\overline{p}(F_f)=(p(F_1),\ldots p(F_f),\ldots p(F_k)) \quad (20)$$

there are any additional top functions which were not relevant for the former product the portability becomes $p(F_j) = 0$.

The vector given in Eq. (20) shows the tendency whether prior information could be considered or not. The value to what extent the information is transferable may be estimated by a weighting of the top functions. In this context, the occurrence of a failure cause is of interest. Every top function can be weighted by:

$$w(F_f)=\frac{\sum_{i=1}^{l}(10-o_{ji})}{\sum_{j=1}^{k}(\sum_{i=1}^{l}(10-o_{ji})} \quad (21)$$

The weighting over all top functions yields a vector

$$\overline{w}(F_f)=(w(F_1),\ldots w(F_f),\ldots w(F_k)) \quad (22)$$

As a result, the decrease-factor is calculated by the scalar product of the two vectors given with Eq. (20) and (22)

$$\delta=\overline{w}(F_f)\overline{p}(F_f)^T \quad (23)$$

The next step of the research activities will be to develop a procedure for the estimation of the decrease-factor regarding prior information given from preceding tests under different test conditions.

EXEMPLARY APPLICATION

The procedures described in this paper are to be presented by means of a synthetic example. The sample-size necessary to demonstrate the product requirements has to be determined. The product reliability $R(t_s)$ = 90% at the specified lifetime t_s = 150 is to be proven with a confidence level C = 90%. The classical theory yields a sample-size of 21 if a uniform distribution is considered as prior knowledge and no failures occur during the test. This number is to be reduced with the use of additional knowledge that is supposed to be available from a fatigue damage calculation, preceding tests, and the failure behavior of a former product.

It is assumed that a fatigue damage calculation yields a reliability of $R(t_{cal})$ = 99% at a lifetime of t_{cal} = 43. On the assumption that the failure behavior is described by means of a Weibull distribution with a shape parameter b = 1.3, the reliability at the desired lifetime is derived from Eq. (13) with $R_{cal}(t_s)$ = 95%. Eq. (14), (5) and (6) yield the posterior probability density function which results in the following equation for the confidence level:

$$C = \frac{\int_{0.9}^{0.95} \frac{0.9}{0.95} R^n dR + \int_{0.95}^{1} \frac{0.1}{0.05} R^n dR}{\int_{0}^{0.95} \frac{0.9}{0.95} R^n dR + \int_{0.95}^{1} \frac{0.1}{0.05} R^n dR} = 0.9 \qquad (24)$$

The necessary sample-size is determined by Eq. (24) as n_{cal} = 17. The information from the calculation reduces the necessary sample-size by Δn_{cal} = 4 items. The sample-size reduction obtained with the calculation can be determined by the following formula:

$$\Delta n_{cal} = \frac{\ln(1-C)}{\ln R(t_s)} - 1 - n_{cal} \qquad (25)$$

It is assumed that prior information is available from a preliminary test. The test is performed with acceleration and a test time which is not equivalent to the specified product lifetime. The test conditions are summarized in Table 1. The parameters of the prior beta probability density function can be calculated by Eq. (10) which yields A_{01} = 6.28 and B_{01} = 0.72.

Table 1: Test conditions

sample-size n0	6
number of failures x_0	0
lifetime-ratio Lr0	0.5
acceleration factor χ_0	3
shape parameter (assumed) b	1.3

Subsequently, it is assumed that failure data of a former product is available from use operation. The data is analyzed by means of a Weibull distribution where the parameters are given in Table 2. The Weibull parameters and Eq. (12) yield the parameters of the prior beta probability density function A_{02} = 70.25 and B_{02} = 5.75

Table 2: Weibull parameters

shape parameter (assumed) b	1.3
scale parameter T	1100
location parameter t0	0
population size npop	75

The sample-size necessary to demonstrate the product requirements has to be determined using all prior information that is available. For multiple sources of prior information the confidence level may be determined by:

$$C = \int_{R(t_a)}^{1} \frac{1}{\beta(A,B)} R^{A-1} (1-R)^{B-1} dR$$

$$A = \sum_{i=1}^{r} \delta_i A_{0i} + 1 + \Delta n_{cal} + n - x,$$

$$B = \sum_{i=1}^{r} \delta_i (B_{0i} - 1) + 1 + x, \qquad (26)$$

The information from the calculation is considered by the sample-size reduction which is calculated by Eq. (25). If the reliability demonstration test is planned by means of a test without the occurrence of a failure, the confidence level may be found from Eq. (26) with x = 0. The decrease-factor δ_1 regarding the preliminary test results is to be estimated with a value between 0 and 1. Figure 4 shows the confidence level as a function of the sample-size for different values of the decrease-factors. The confidence level increases with higher values of the decrease factors and higher sample-sizes. According to Figure 4, the confidence level reaches the lowest values if the given prior information is not considered. In this case, only the uniform distribution is used as prior knowledge. If the decrease factors are chosen with $\delta_1 = \delta_2 = 0$, only the prior information given from the calculation is used for the planning of the reliability demonstration test. The confidence level is higher than the required 90% for every value of the sample-size, if the decrease- factors are chosen with $\delta_1 = \delta_2 = 1$.

Figure 5 shows the sample-size necessary to demonstrate the product requirements ($R(t_s)$ = 90%, C = 90%). The sample size is determined by using the calculation results, the results of the preceding test and the knowledge about the failure behavior of the former product. The decrease-factor regarding the preliminary test and the information of the former product is estimated with different values. As can be seen from Figure 5, the sample-size increases with lower decrease-factors. For decrease factors $\delta_1 = \delta_2 = 0$, the necessary sample-size is n = 17. The sample-size can be reduced to n = 0 if all given information is totally transferable ($\delta_1 = \delta_2 =$ 1). For other values of the decrease-factors the sample-size lies between 0 and 17. Compared with the classical method where n = 21 test items are necessary to prove the requirements, it is possible to reduce the sample-size if prior knowledge is considered. The sample-size is reduced by at least 4 test items. For the best case no subsequent test would be necessary. However, due to differences in test conditions or environment and function the total use of prior information is not recommended. To minimize the risk of failing to meet the product requirements the decrease-factor should be estimated with lower values than 1.

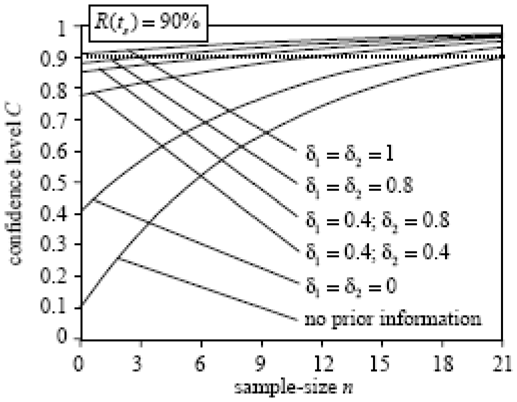

Figure 4: Confidence level as a function of the sample-size for different decrease-factors

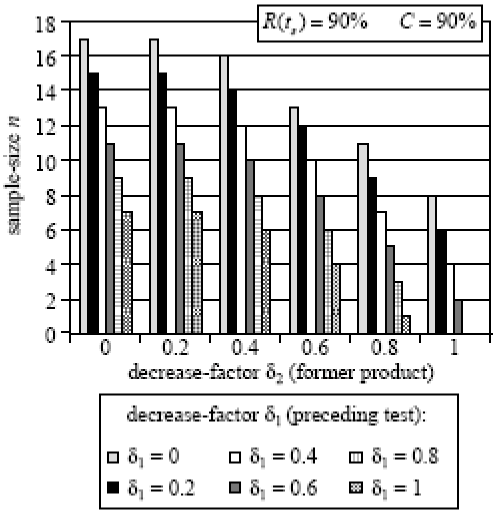

Figure 5: Necessary sample-size for demonstrating the product requirements

The higher the reliability requirements on a product are the more extensive is the test for proving the reliability targets. The classical theory to determine sampling plans yields a large sample-size necessary to demonstrate the product reliability Above all, the sample-size increases, if failures have to be taken into account. To use all information about the product given through the development process the application of Bayes procedure is recommended. The reliability demonstration test can be planned optimally regarding sample-size and test duration, if information from product development is utilized. In this case the random variable corresponds to the product reliability at the specified lifetime in the field. Afterwards, the posterior distribution of the reliability is generated with the actual sample distribution by means of Bayes procedure. Here, the problems arise on the one hand in the definition of the prior distribution and on the other hand in the different sources of information that are used within the planning of the reliability demonstration test.

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