About Eddy Currents in Induction Melting Processes

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***Abstract* – In this paper we present a method for computing the eddy currents in induction melting processes for non-ferrous alloys. We take into consideration the situation when only the crucible is moving, inside the coils. This fact makes differential computation methods to be hard to apply, because is necessary to generate a new mesh and a new system matrix for every for every new position of the crucible related to the coils. Integral methods cancel this drawback because the mesh is generated only for the domains with eddy currents. For integral methods, the mesh and the inductance matrix remain unchanged during the movement of the crucible; only the free terms of the equation system will change.**

Keywords: ***eddy current, induction, heating process, melting.***

I. INTRODUCTION

In 2D structures, the numerical solving of the integral equation of the current density is done dividing the conducting domains into polygonal sub-domains (rectangles). For each sub-domain the current density is considered to be constant (volume elements) [5, 6, 7].

A comparative analysis of differential and integral computation methods for the eddy currents problem is presented in [3] and more recently in [4]. They use two dual differential methods (**A-Φ**) and (**T-Ω**) (it is also suggested a method to determine the constitutive error in the solution field, useful for the accuracy of the result) and one integral method that reduce the nonlinear field problem to solving a nonlinear equivalent system. When using the integral method for problems in which we have to determine precisely the distribution of eddy currents and thee losses, we get a new computation method. Using **T** as unknown, an integral formulation and a topological standardization condition, [1] presents the eddy currents problem in conducting environments. The used method has the advantage that only the conducting domains are meshed, thus the number of unknowns is lower, equal to the number of internal edges of the cotree associated to the mesh graph, and the system matrix is inverted only once. Although the system matrix is full, it has small dimensions. The sources are aerial coils passed through by time varying currents and the resulted field is computed using the Biot-Savart formula. This method is easily adapted to multi-connected domains; no cuts are necessary [2]. When having multi-connected domains, the condition $\mathbf{T}\times\mathbf{n} = 0$ associated to the choice of active edges starting from an unfolded tree in the conducting environment meshed with volume elements, becomes too restrictive when we also have flow currents. We have to add a certain number of degrees of freedom equal to the number of cuts necessary to reduce the R^3-Ω domain to a simple-connected domain. These additional degrees of freedom can be associated with a set of line integrals for **T** along the edges that intersect the cutting surfaces necessary in the classic method. We use a tree that we generate starting from the boundary edges, thus forming a complete tree. Let **E** be the number of boundary edges and V-1 the number of boundary nodes, the number of active boundary elements is E-V+1.

For nonlinear problems the differential methods seem less limited, as long as for applying the Biot-Savart formula to compute the field with an integral method, we have to assume the magnetic homogeneity of the space, this being impossible for applications that have domains with high magnetic permeability and also small air gaps.

II. AN INTEGRAL FORMULATION OF THE EDDY CURRENTS PROBLEMS IN 3D STRUCTURES

Let $\mathbf{J}_0$ be the imposed current density. The superconductor coils have an imposed value for $\mathbf{J}_0$ that has to be the same in cross-section. Let **J** be the eddy currents density. This can be found in all conducting material that has no coils. The entire space has the magnetic permeability of the void μ_0.

The reference system is set to the moving parts. From Faraday's law we have:

$$\mathbf{E} = -\left(\frac{\partial \mathbf{A}}{\partial t} + gradV\right) \quad (1)$$

We consider the environment to be linear and magnetically homogenous, but has the magnetization **M**, useful when non-linear environments Ω_F are computed using the polarization method:

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$$

Then the following equation results:

$$rotrot\mathbf{A} = \mu_0\left(\mathbf{J} + \mathbf{J}_0 + rot\mathbf{M}\right)$$

where the solution for the entire space is given by the Biot-Savart-Laplace formula:

$$\mathbf{A} = \frac{\mu_0}{4\pi}\int_{\Omega}\frac{\mathbf{J}}{r}dv + \mathbf{A}_0 \tag{2}$$

where Ω is the domain of the eddy currents, $\mathbf{A}_0$ is the vector potential produced by the imposed current density $\mathbf{J}_0$ and magnetization **M** (Fig. 2)

$$\mathbf{A}_0 = \frac{\mu_0}{4\pi}\int_{\Omega_0}\frac{\mathbf{J}_0}{r}dv + \frac{\mu_0}{4\pi}\int_{\Omega_F}\frac{rot\mathbf{M}}{r}dv \tag{3}$$

if we apply a partial integral, the last part of the right term of the equation becomes:

$$\int_{\Omega_F}\frac{rot\mathbf{M}}{r}dv = \int_{\Omega_F} rot\frac{\mathbf{M}}{r}dv + \int_{\Omega_F}\frac{\mathbf{M}\times\mathbf{r}}{r^3}dv$$

$$= \oint_{\partial\Omega_F}\mathbf{n}\times\frac{\mathbf{M}}{r}dS + \int_{\Omega_F}\frac{\mathbf{M}\times\mathbf{r}}{r^3}dv$$

If we keep the boundary $\partial\Omega_F$ of domain Ω_F outside the parts with magnetization, then:

$$\int_{\Omega_F}\frac{rot\mathbf{M}}{r}dv = \int_{\Omega_F}\frac{\mathbf{M}\times\mathbf{r}}{r^3}dv \tag{4}$$

From equations 1-4 and from the law of conduction we obtain the integral equation of eddy currents:

$$\rho\mathbf{J} + \frac{\mu_0}{4\pi}\frac{d}{dt}\int_{\Omega_C}\frac{\mathbf{J}}{r}dv + gradV =$$

$$-\frac{\mu_0}{4\pi}\frac{d}{dt}\int_{\Omega_0}\frac{\mathbf{J}_0}{r}dv - \frac{\mu_0}{4\pi}\frac{d}{dt}\int_{\Omega_F}\frac{\mathbf{M}\times\mathbf{r}}{r^3}dv \tag{5}$$

where ρ is the rezistivity in the Ω domain.

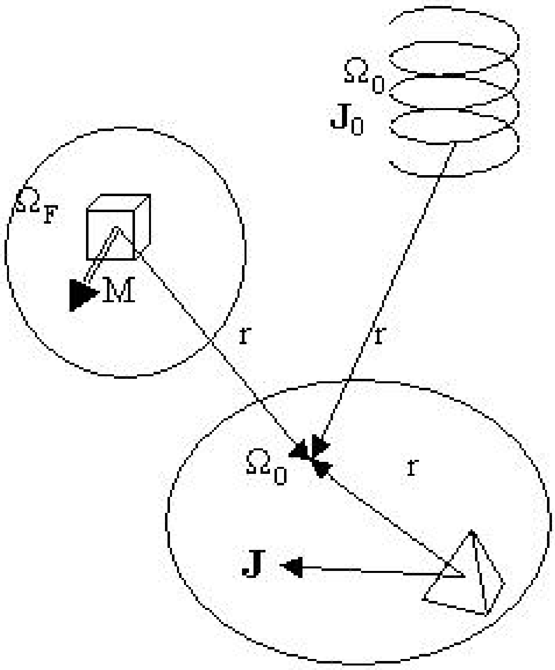

Fig. 1.

The magnetic circuit law imposes a scale condition for **J**: div **J** = 0. This condition is fulfilled by introducing the electric potential vector **T** as:

$$rot\,\mathbf{T} = \mathbf{J} \tag{6}$$

On the ∂Ω boundary we have the condition:

$$\mathbf{J}\,\mathbf{n} = \mathbf{n}\,\mathrm{rot}\,\mathbf{T} = 0 \tag{7}$$

Equations 6 and 7 cannot define uniquely the potential **T**. We need to add a scale condition for **T**.

Using the Galerkin technique, we consider $\mathbf{N}_k$ n functions defined on Ω having $rot\mathbf{N}_k$ linearly independent. We consider:

$$\mathbf{T} = \sum_{k=1}^{n}\alpha_k(t)\mathbf{N}_k \tag{8}$$

In order to fulfill equation 7, we impose that the tangential component of $\mathbf{N}_k$ to the boundary ∂Ω is null. Projecting equation 5 on the vectors $rot\mathbf{N}_k$ we obtain:

$$\int_{\Omega}\rho\cdot rot\mathbf{T}\cdot rot\mathbf{N}_k dv + \frac{d}{dt}\int_{\Omega}\int_{\Omega}\frac{1}{r}\cdot rot\mathbf{N}_k\cdot rot\mathbf{T}dvdv$$

$$= -\int_{\Omega} rot\mathbf{N}_k\cdot\frac{\partial\mathbf{A}_0}{\partial t}dv$$

k=1,2,..,n (9)

Applying a partial integration and taking into account that $\mathbf{N}_k$ has a null tangential component on ∂Ω gives us:

$$0 = \int_{\Omega} rot\mathbf{N}_k\cdot gradVdv =$$

$$\oint_{\partial\Omega}(\mathbf{n}\times\mathbf{N}_k)gradV\,dS + \int_{\Omega}\mathbf{N}_k rot(gradV)\,dv$$

equation (9) cam also be written:

$$\{R\}\,[I] + \frac{d\{L\}[I]}{dt} = -\frac{d}{dt}[\Phi] \tag{10}$$

where

$\{I\} = (\alpha_1, \alpha_2, \ldots, \alpha_n)^T$,

$\{\Phi\} = (\Phi_1, \Phi_2, \ldots, \Phi_n)^T$

$\Phi_k = \int_{\Omega} rot\mathbf{N}_k\mathbf{A}_0 dv$,

$R_{ik} = \int_{\Omega}\rho\, rot\mathbf{N}_i rot\mathbf{N}_k dv$,

$L_{ik} = \frac{\mu_0}{4\pi}\int_{\Omega}\int_{\Omega}\frac{1}{r}rot\mathbf{N}_j rot\mathbf{N}_k dvdv$,

the initial conditions for equation 10 result from **J** = rot**H**.

For structures with moving parts, the L_{ik} coefficients are time varying when they are assigned to conducting sub-domains with different speeds, because r is time varying. If the structures do not move, than matrix $\{L\}$ from equation 10 is invariable in time and quits the derivate:

$$\{R\}\,[I] + \{L\}\frac{d[I]}{dt} = -\frac{d}{dt}[\Phi]$$

3. EDGE ELEMENTS

We divide domain Ω into ω_i sub-domains. For simplicity, we assume that every sub-domain is tetrahedral. In each tetrahedron we assume **T** to be:

$$\mathbf{T} = \mathbf{a} + \mathbf{b}\times\mathbf{R} \tag{11}$$

Knowing the line integrals of **T** on the tetrahedrons' edges, then **T** is well defined wherever inside the tetrahedrons:

$$\mathbf{a} = \frac{\tau_1(R_2\times R_3)+\tau_2(R_3\times R_1)+\tau_3(R_1\times R_2)}{6v} =$$

$$-\frac{\tau_1 S_1+\tau_2 S_2+\tau_3 S_3}{3v}$$

$$\mathbf{b} = \frac{(\tau_2-\tau_3+\tau_{23})R_1+(\tau_3-\tau_1+\tau_{31})R_2+(\tau_1-\tau_2+\tau_{12})R_3}{6v}$$

where τ_i and τ_{ij} are the line integrals of T on the $P_0\,P_i$ and $P_i\,P_j$ edges, v is the volume of the tetrahedron and S_i is the oriented surface of the tetrahedron, (fig 2).

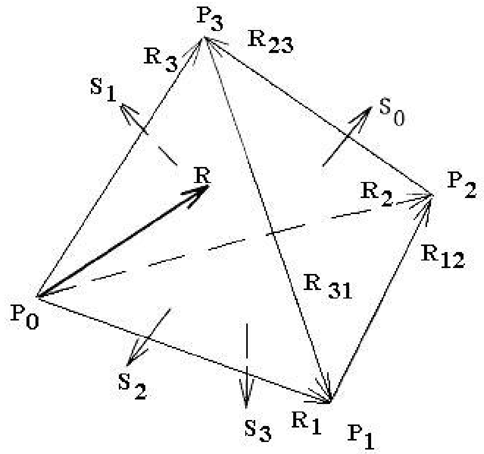

Fig.2.

We consider function $\mathbf{N}_k$, whose behavior is imposed by equation (11) in every tetrahedron and has unitary edge values on the k edge and null edge values for every edge with $i \neq k$;

$$\mathbf{N}_k = \begin{cases} \ldots\ldots\ldots \\ -\dfrac{\mathbf{S}_{i_1}}{3v_i}+\dfrac{\mathbf{R}_{i_{23}}}{6v_i}\times\mathbf{R} & \text{if } P\in\omega_i \\ \ldots\ldots\ldots \end{cases}$$

where i is the index of the ω_i tetrahedron having the k edge.

Remark 1. If edge k is oriented pointing from P_i to P_j and W_i, W_j are the nodal elements of points i and j, then:

$\mathbf{N}_k = W_i\, grad\, W_j - W_j\, grad\, W_i$

Remark 2. We can easily notice that:

a) *div* **T** = 0 inside the tetrahedrons,

b) *rot* **T** = 2**b**,

c) the tangent component of **T** is preserved on the separation surfaces,

d) the normal component of **T** is not preserved on the separation surfaces,

e) the normal component of **J** = *rot* **T** is preserved on the separation surfaces. Thus, the volume divergence of **J**, as well as the specific divergence of **J** is zero everywhere,

f) on a surface bordered by edges k, l, m, the flux of **J** is given by the contour integral of **T** on the k, l, m edges.

Lets consider the graph made up of the edges of a finite elements network using tree-cotree decomposition. Obviously, the closed contour integral of **T** defines uniquely the flux of **J** on the surface bordered by the contour. But, the fluxes of **J** cannot define uniquely the edge values of **T**. We know from the circuit theory that we can add any value on the edge of the tree if we also add values for the cotree's edges, values that verify the second Kirchhoff law. So, we can cancel the values on the tree's edges. Thus, the fluxes of **J** on the surfaces define the values on the **T** cotree's edges, and these values are unique. The above-mentioned topological condition is a standardization condition that assures the uniqueness of the values on the edges of **T**. These values are imposed null on the tree's edges and can be not null on the cotree's edges. Thus, the number of edges of the cotree gives the degrees of freedom of **T**.

The boundary condition (7) cancels any contour integral on any closed contour of **T** along the $\partial\Omega$ boundary. So, first we have to define the boundary tree, and only after that, the tree for the inside edges of the Ω domain. These values on the boundary cotree's edges are null.

For multiple-connective domains, we cannot cancel all the integrals of **T** to the $\partial\Omega$ boundary. We have γ loops that surround the tube, a current i can run through this tube (fig. 3), and the integrals of **T** on these contours cannot be canceled. We have a system *{k}* of branches of the cotree on any $\partial\Omega_k$ surface that encloses loops in the same Ω_k domain.

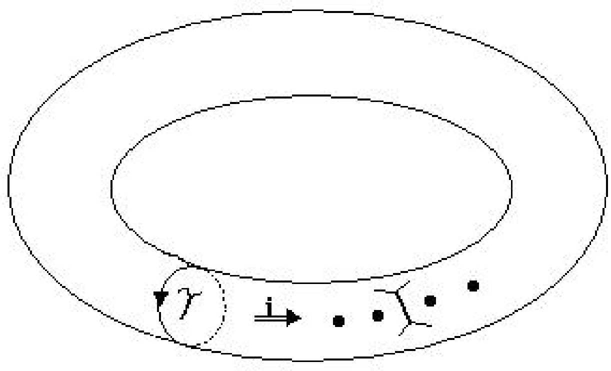

Fig. 3.

All the edges of the cotree belonging to the system *{k}* have the same (unknown) value on the edge.

Lets consider $\mathbf{F}_k$ to be the functions having unitary edge values for all the edges belonging to the system *{k}* and null values on the other edges:

$$\mathbf{F}_k = \sum_{i\in\{k\}} \mathbf{N}_i$$

Thus, from (6) we get:

$$\mathbf{T} = \sum_{j=1}^{n_i} \alpha_j(t)\mathbf{N}_j + \sum_{k=1}^{n_b} i_k(t)\mathbf{F}_k \qquad (8')$$

where n_i is the number of edges of the cotree from inside the Ω domain and n_b is the number of boundary edge systems. From (10) we can conclude:

$I = (\alpha_1,\alpha_2,\ldots,\alpha_{ni},i_1,i_2,\ldots,i_{nb})T$

When using first-degree edge elements on the tetrahedral sub-domains, we get:

$$R_{ik} = \sum_{p\in\{i\}\cap\{k\}} rot\mathbf{N}_{i_p}\, rot\mathbf{N}_{k_p}\, V_p ,$$

where $p \in \{i\} \cap \{k\}$ is the index for the ω_p sub-domains that contain the *i* and *k* edges, and $rot\mathbf{N}_{i_p}$ is the value of $rot\mathbf{N}_i$ in the ω_p sub-domain.

$$L_{ik} = \frac{\mu_0}{4\pi} \sum_{p\in\{i\}} \sum_{q\in\{k\}} rot\mathbf{N}_{i_p} rot\mathbf{N}_{k_q} \int_{\omega_p} \int_{\omega_q} \frac{1}{r} dv dv$$

When we have magnetically nonlinear environments, is better for the magnetization M to be constant in the ω_l sub-domains that compose the ferromagnetic domain Ω_F. These sub-domains can have any form (volume elements). Thus, Φ_k becomes:

$$\Phi_k = \Phi_{Fk} + \Phi_{J_k}$$

where

$$\Phi_{F_k} = \sum_{l=1}^{n_F} \mathbf{a}_{kl} \mathbf{M}_l$$

and

$$\mathbf{a}_{kl} = -\frac{\mu_0}{4\pi} \sum_{p\in\{k\}} rot\mathbf{N}_{k_p} \times \int_{\omega_p} \int_{\omega_l} \frac{\mathbf{r}}{r^3} dv \tag{12}$$

or

$$\mathbf{a}_{kl} = -\frac{\mu_0}{4\pi} \sum_{p\in\{k\}} rot\mathbf{N}_{k_p} \times \int_{\omega_p} \oint_{\partial\omega_l} \frac{\mathbf{n}}{r} dS \tag{12'}$$

If we also consider $\mathbf{J}_0$ to be constant in the ω_z sub-domains, then:

$$\Phi_{J_k} = \sum_{l=1}^{n_0} \mathbf{b}_{kz} \mathbf{J}_{0_z}$$

where

$$\mathbf{b}_{kz} = \frac{\mu_0}{4\pi} \sum_{p\in\{k\}} rot\mathbf{N}_{k_p} \int_{\omega_p} \int_{\omega_z} \frac{1}{r} dv \tag{13}$$

or

$$\mathbf{b}_{kz} = -\frac{\mu_0}{8\pi} \sum_{p\in\{k\}} rot\mathbf{N}_{k_p} \int_{\omega_p} \oiint_{\partial\omega_z} \frac{\mathbf{nr}}{r} dS \tag{13'}$$

Remarks:

1. If sub-domains ω_k and ω_l are far from each other, is better to use equation (12) which, for distances long enough, can be replaced by:

$$\mathbf{b}_{kz} = \frac{\mu_0}{4\pi} \sum_{p\in\{k\}} rot\mathbf{N}_{k_p} \frac{1}{r_{pl}} V_p V_l ,$$

r_{pl} is the distance between the weight centers ω_k and ω_l sub-domains.

2. If sub-domains ω_k and ω_l are close enough, is better to use equation (12').

3. If we chose polyhedral sub-domains, then we can write in (12'):

$$\oiint_{\partial\omega_l} \frac{\mathbf{n}}{r} dS = \sum_{f\in\{l\}} \mathbf{n}_f \int_{S_f} \frac{1}{r} dS$$

Remarks 1,2,3 are also valid for equation (13) and (13').

For magnetically nonlinear environments is necessary to determine the magnetic induction **B** that later adjusts the value of the magnetization fulfilling this equation:

$$\mathbf{M} = \frac{1}{\mu_0} \mathbf{B} - \hat{\mathbf{H}}(\mathbf{B})$$

Knowing the current density **J** and the magnetization **M**, from the Biot-Savart-Laplace formula we extract the average value of the magnetic induction in the ω_i sub-domain:

$$\tilde{\mathbf{B}}_i = \frac{1}{v_i} \sum_{k=1}^{n} \boldsymbol{\beta}_{ik} \times \mathbf{J}_k - \frac{1}{v_i} \sum_{p=1}^{n_F} \overline{\overline{\gamma}}_{ip} \mathbf{M}_p + \mathbf{B}_0$$

where:

$$\boldsymbol{\beta}_{ik} = -\frac{\mu_0}{4\pi} \int_{\omega_i} \int_{\omega_k} \frac{\mathbf{r}}{r^3} dv_i\, dv_k = \frac{\mu_0}{4\pi} \oiint_{\partial\omega_i} \int_{\omega_k} \frac{\mathbf{n}_i}{r} dv_k dS_i$$

$$= \frac{\mu_0}{8\pi} \oiint_{\partial\omega_i} \oiint_{\partial\omega_k} \frac{(\mathbf{r}_{ik}\, \mathbf{n}_k)\mathbf{n}_i}{r} dS_p dS_i$$

and:

$$\overline{\overline{\gamma}}_{il} = \frac{\mu_0}{4\pi} \oint_{\partial\omega_l} \oint_{\partial\omega_l} \frac{\mathbf{n}_l ; \mathbf{n}_i - (\mathbf{n}_l \cdot \mathbf{n}_i)\overline{\overline{1}}}{r} dS_l dS_i$$

because, assuming that the magnetization has constant values in the ω_l sub-domains, we get:

$$\tilde{\mathbf{B}}_i = \frac{1}{\mathrm{v_i}} \frac{\mu_0}{4\pi} \int_{\omega_i} \int_{\Omega_F} \frac{rot\mathbf{M} \times \mathbf{r}}{r^3} dv_i\, dv =$$

$$-\frac{1}{\mathrm{v_i}} \frac{\mu_0}{4\pi} \sum_{l=1}^{n_F} \int_{\omega_i} \oiint_{\partial\omega_l} \frac{(\mathbf{n}_l \times \mathbf{M}_l) \times \mathbf{r}}{r^3} dS_l\, dv_i =$$

$$= \frac{1}{\mathrm{v_i}} \frac{\mu_0}{4\pi} \sum_{l=1}^{n_F} \oiint_{\partial\omega_i} \oiint_{\partial\omega_l} \frac{(\mathbf{n}_l \times \mathbf{M}_l) \times \mathbf{n}_i}{r} dS_i\, dS_l =$$

$$\frac{1}{\mathrm{v_i}} \frac{\mu_0}{4\pi} \sum_{l=1}^{n_F} \oiint_{\partial\omega_i} \oiint_{\partial\omega_l} \frac{(\mathbf{n}_l \mathbf{n}_i)\mathbf{M}_l - \mathbf{n}_l (\mathbf{n}_i \mathbf{M}_l)}{r} dS_i\, dS_l =$$

$$\frac{1}{\mathrm{v_i}} \frac{\mu_0}{4\pi} \sum_{l=1}^{n_F} \oiint_{\partial\omega_i} \oiint_{\partial\omega_l} \frac{(\mathbf{n}_l \mathbf{n}_i)\overline{\overline{1}} - (\mathbf{n}_l ; \mathbf{n}_i)\mathbf{M}_l}{r} dS_i\, dS_l ,$$

We made use of the fact that the rot operator has not null values only on the boundaries of the ω_l sub-domains ($rot\mathbf{M} = \mathbf{n}_{1,2} \times (\mathbf{M}_2 - \mathbf{M}_1)$).

$\mathbf{B}_0$ is the magnetic induction produced by the $\mathbf{J}_0$ imposed currents. This value is computed once, at the beginning of the iterations.

The time integration of equation (10) can be done assuming that **J** varies linearly in Δt_m time intervals. After the integration of (10) on this time interval we get:

$$\frac{1}{2}\{R\}\ ([I_m] + [I_{m-1}])\Delta t_m +$$
$$+\ \{L_m\}[I_m] - \{L_{m-1}\}[I_{m-1}]$$
$$= -[\Phi_m] + [\Phi_{m-1}]$$

Knowing the value for J at t_{m-1} (I_{m-1}), from the above-mentioned equation we get the value of for t_m (I_m).

4. CONCLUSIONS

We presented here the results of a melting process of a non-ferrous alloy using a graphite crucible.

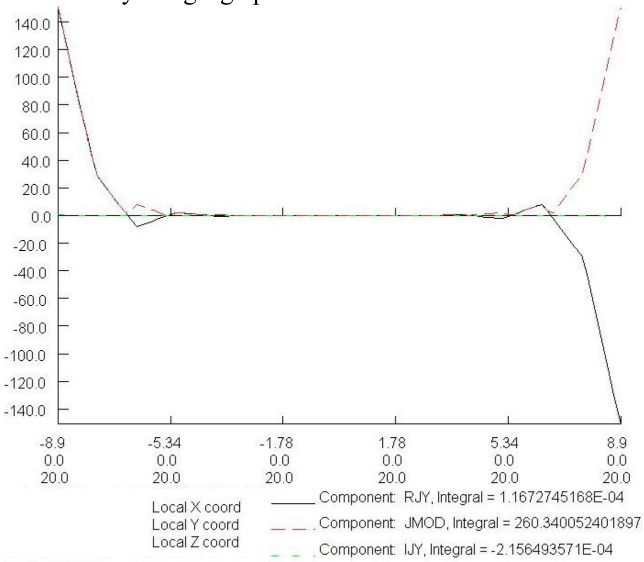

Fig. 4.

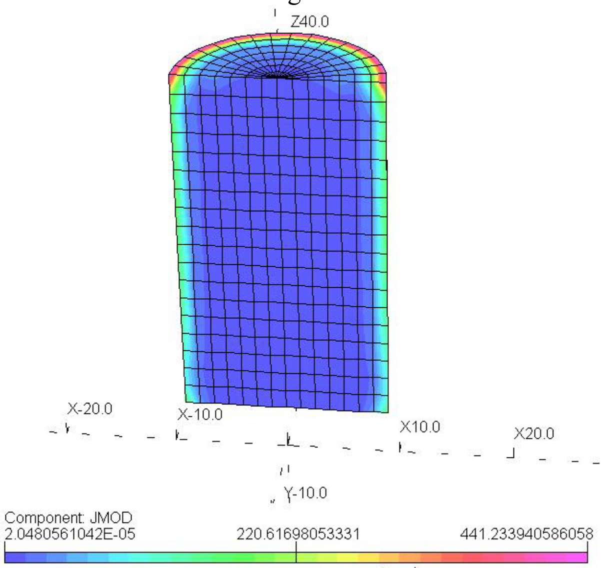

Fig. 5.

Figure 4 presents the variation of imaginary J, real J and module of J - the current density on the X,Y and Z axis. Figure 5 presents the variation of module J.

We presented a computation method of the integral equation of the current density in 3D structures. For these 3D structures we used the Biot-Savart-Laplace formula. For the numerical solving we used electric potential vector that assures a null divergence for **J** and which can easily define the boundary condition for **J** on the surfaces of conducting parts (**J_n=0**). We presented a method to determine the electric potential vector, method that uses edge elements from the edge graph of a mesh.

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