Gyrators in electrical power management systems

St. Hărăguş[*], D. Toader[*], I. Tatai[*]

[*]Department of Physical Foundation of Engineering
"Politehnica" UniversityTimişoara,
Postal address: Timişoara, B-l V. Pârvan nr.2,
e-mail: stefan.haragus@et.upt.ro

***Abstract** – The paper analyses two main types of gyrator-like dc-dc switched converters encountered in dc power management. The first type consists of a reactive reciprocal two-port network inserted between two bridges of ideal switches. The power transfer from source to load is controlled by modifying the delay between the switching bridges. The second type is a buck converter with an appropriate sliding mode control, suitable for paralleling multiple converters.*

***Keywords:** nonreciprocal circuits, gyrators, dc switching converters.*

I. INTRODUCTION

The gyrator as an ideal, lossless nonreciprocal two-port circuit element has been introduced by the Dutch engineer B. D. H. Tellegen [1]. The main property of the gyrator, namely the conversion of the network connected at the output port into its dual, as seen from the input port, was supposed by Tellegen to help synthesizing various electrical filters in the field of signal processing.

There are many ways to physically implement a gyrator. The first gyrator device ever built was based on the Faraday rotation effect, and it operates in the range of microwaves [2]. In the quest for physical gyrators it has been discovered that the Hall effect can be used to build gyrators that could operate in a very broad range of frequencies [3]. Unfortunately, Hall effect based gyrators do not imposed as inductance emulators due to their large ohmic dissipation. Practical applications of the gyrator at low frequencies occurred with the development of active semiconductor device technology, especially due to the need to integrate large values of inductances. Based on transistors and operational amplifiers, many practical gyrator circuits have been proposed and used [4].

All the gyrator realizations mentioned so far were used in the signal processing technique and operates time-continuously. On the other hand, in power management some dc-dc switching converters that have a gyrator-like behavior are being used. Thus, a switched double-bridge dc-dc converter behaves as a dc gyrator, as long as the mean values of the voltages and currents at the ports are considered [5]. Another dc gyrator form the same class, based on a switched λ/4 transmission line, has also been reported [6]. A different class of gyrator-like dc-de converters makes use of the sliding-mode control [6, 7, 11]. In contrast to the double-bridge converter, which has a natural dc gyrator behavior, the second type of converters is forced to behave like a dc gyrator by a special control loop.

In this paper the dependence of two-port parameters of the double-bridge gyrator on the network's parameters as well as on the switching delay between the bridges will be analyzed. Also, the configuring of a buck converter as a gyrator using sliding-mod control will be explained.

II. BASIC PROPERTIES OF ELECTRICAL GYRATORS

An antireciprocal two-port circuit is a special case of a nonreciprocal two-port circuit whose parameters satisfy a certain condition, usually referred as the antireciprocity condition. For example, if the two-port's equations are expressed via the short-circuit admittances,

$$\begin{aligned} \underline{I}_1 &= \underline{Y}_{11}\underline{U}_1 + \underline{Y}_{12}\underline{U}_2 \\ \underline{I}_2 &= \underline{Y}_{21}\underline{U}_1 + \underline{Y}_{22}\underline{U}_2 \end{aligned}, \tag{1}$$

then the antireciprocity condition is [8]

$$\underline{Y}_{12} = -\underline{Y}_{21}, \tag{2}$$

where the passive rule for U and I at both ports has been assumed (Fig.1).

The physical meaning of eq.(2) is as follows: if a voltage source, connected at the input port, determines a current $\underline{I}$ at the short-circuited output port then, the same voltage source, if connected at the output port, will determine, at the short-circuited input port, a current of the same amplitude but of oposed phase, -$\underline{I}$. A similar antireciprocity condition results if the impedance formalism for the two-port is used:

$$\underline{Z}_{12} = -\underline{Z}_{21}. \tag{3}$$

For an ideal gyrator, which is a lossless passive antireciprocal two-port network, eqs. (1) become

$$\begin{aligned} \underline{I}_1 &= \pm g \underline{U}_2 \\ \underline{I}_2 &= \mp g \underline{U}_1 \end{aligned}, \tag{4}$$

where parameter g is given the name gyration conductance and is a constitutive parameter of the gyrator. The circuit symbol for a gyrator is shown in Fig.1.

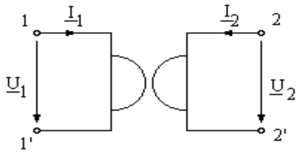

Fig.1. Circuit symbol for a gyrator

Eqs. (4) reveal the two basic properties of an ideal gyrator. Thus, if the gyrator is loaded by impedance $\underline{Z}_s$, then the input impedance is given by equation

$$\underline{Z}_{in} = r^2 \frac{1}{\underline{Z}_s}, \tag{5}$$

where $r=g^{-1}$ is the gyration resistance. This property allows emulating large inductances at low frequencies by using a capacitivelly loaded gyrator. On the other hand, from eqs. (4) also follows that

$$P_1 = -P_2, \tag{6}$$

P_1 and P_2 being the active power at the input, respectively the output port of the gyrator. With the assumed passive rule at both ports, this means that the active power supplied at one port will be recovered at the other port. Hence, ideal gyrators belong to a larger power processing circuits called POPI (power output equals power input).

For power processing circuits not only the POPI feature is of relevance, but also is important to efficiently control the mean power transferred from one port to the other. In the case of gyrators as power management circuits, this is achieved by modifying the gyration conductance.

Switched gyrators are also described by two-port equations with the difference that the currents and voltages occurring in these equations represent the mean values of the instantaneous variables taken over a switching period. Hence, the two-port equations of a lossy switched gyrator can be expressed, for example, in the form

$$\begin{aligned} I_1 &= G_{11} U_1 + G_{12} U_2 \\ I_2 &= G_{21} U_1 + G_{22} U_2 \end{aligned}. \tag{7}$$

Therefore, switched gyrators can be viewed as dc gyrators.

An equivalent circuit can be associated to eqs. (7), as shown in Fig. 2.

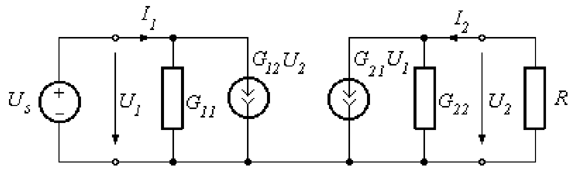

Fig.2. Current sources equivalent circuit of a lossy gyrator.

A dual equivalent circuit is shown in Fig.3, if the R_{ij} parameters are used to describe the gyrator.

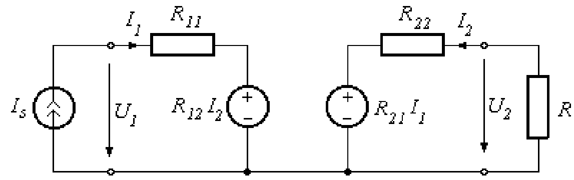

Fig.3. Voltage sources equivalent circuit of a lossy gyrator.

The equivalent circuit in Fig.2 clearly shows that a voltage source connected at the input port is seen by the load connected at the output port as a current source. In distributed power systems often dc-dc converters have to be parallel interconnected, as paralleling increase the processing capability and improve the systems reliability. Current sources are very suitable for paralleling and this seems to be the main reason to configuring a converter as a gyrator.

III. DC-DC CONVERTERS WITH NATURAL GYRATOR-LIKE BEHAVIOR

Maybe the simplest switched gyrator is the double bridges d.c.-d.c. converter shown in Fig.4.

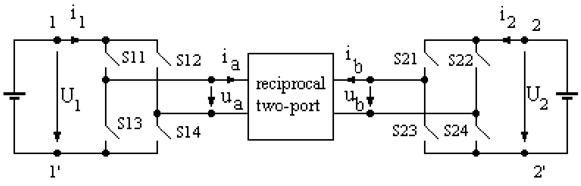

Fig.4. Schematics of the double-bridge converter

It consists of two switching bridges, linked by a reactive reciprocal two-port network, used for temporary storage of energy.

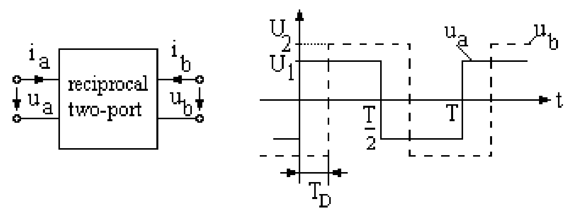

Fig.5. The storage network of the converter

The switching sequence for the input bridge is as follows: first, S_{11} is closed synchronously with S_{14} then, after a time interval of $T/2$, S_{12} and S_{13} are closed and, at the same moment, S_{11} and S_{14} are opened; after a time interval of $T/2$ the sequence starts again and so on. The switches of the output bridge follows the same switching sequence as the input bridge, but delayed by a time interval T_D. Further, we will assume that at the converter ports are connected two dc voltage sources, of magnitudes U_1, respectively U_2.

Therefore, assuming that the switches $S_{11}, S_{12}, \ldots, S_{24}$ are ideal, the ports of the reciprocal two-port network are fed with rectangular voltages, u_a and u_b, with magnitudes U_1, respectively U_2, delayed by a time interval T_D (Fig.5)

Our goal is to derive the dc two-port equations of this converter and the necessary conditions for gyrator-like behavior. The analysis, which is given in details in [9, 10] is based on the Fourier spectral decomposition of the rectangular voltage waveforms of the equivalent sources; this allows expressing the currents $i_a(t)$ and $i_b(t)$ in terms of the reciprocal two-port network parameters. Then, the currents $i_1(t)$ and $i_2(t)$ at the converter's ports can be expressed in terms of i_a and i_b which eventually yields the mean currents I_1 and I_2 by integrating over a period T:

$$\begin{aligned} I_1 &= G_{11}U_1 + G_{12}U_2 \\ I_2 &= G_{21}U_1 + G_{22}U_2 \end{aligned}, \tag{8}$$

where

$$G_{11} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} G_{11}^{(k)}, \tag{9}$$

$$G_{22} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} G_{22}^{(k)}, \tag{10}$$

$$G_{12} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} (G_{12}^{(k)} \cos k\omega T_D - B_{12}^{(k)} \sin k\omega T_D), \tag{11}$$

$$G_{21} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} (G_{21}^{(k)} \cos k\omega T_D + B_{21}^{(k)} \sin k\omega T_D) \tag{12}$$

The storage network being reciprocal,

$$G_{12}^{(k)} = G_{21}^{(k)}, \quad B_{12}^{(k)} = B_{21}^{(k)}, \tag{13}$$

hence eqs.(11, 12) become

$$G_{12} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} G_{12}^{(k)} \cos k\omega T_D - \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} B_{12}^{(k)} \sin k\omega T_D \tag{14}$$

$$G_{21} = \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} G_{12}^{(k)} \cos k\omega T_D + \frac{8}{\pi^2} \sum_{k=1,3,\ldots} \frac{1}{k^2} B_{12}^{(k)} \sin k\omega T_D \tag{15}$$

The dc two-port driving point conductances (eq.9, 10) do not dependent on the time delay T_D. For a lossless reciprocal network $G_{11}=G_{22}=0$ and eqs.(8) correspond to a lossless nonreciprocal two-port:

$$\begin{aligned} I_1 &= G_{12}U_2 \\ I_2 &= G_{21}U_1 \end{aligned}, \tag{16}$$

with G_{12} and G_{21} given by eqs.(14, 15). These transfer conductances depend both on the nature of the reciprocal network elements, as well on the time delay T_D. If the reciprocal network is composed solely of inductances and/or capacitances, then

$$\begin{aligned} G_{11} &= G_{22} = 0 \\ G_{12} &= -\frac{8}{\pi^2} \sum_{1,3,\ldots}^{\infty} \frac{1}{k^2} B_{12}^{(k)} \sin k\omega T_D = -G_{21} \end{aligned} \tag{17}$$

which corresponds to an ideal dc gyrator.

On the other hand, if $T_D = T/4$ then the converter behaves like a gyrator irrespective of elements of the reciprocal network. If $T_D = 3T/4$ then the dc transfer conductances swap signs. This is an important property of the switched gyrator as it can be seen from equations

$$\begin{aligned} P_1 &= U_1 I_1 = G_{12} U_1 U_2 \\ P_2 &= U_2 I_2 = G_{21} U_1 U_2 \end{aligned}. \tag{18}$$

that for a lossless reciprocal network the sign of the transfer conductances determines the direction in which the mean power flows from one port to the other. Not only can the direction of the flow be controlled by T_D but also the magnitude of the transferred mean power. For example, if $T_D=0$ or $T/2$, there is no mean power transfer from one port to the other. A net power transfer takes place if $T_D \in (0, T/2)$ or $T_D \in (T/2, T)$. If T_D belongs to the first half period, the mean power is transferred from the input port to the output port, whereas for T_D belonging to the second half period results in an opposite power transfer direction. It is conjectured that for a given network, the maximum mean power transferred from one port to the other, occurs when $T_D=T/4$, respectively $T_D=T/2+T/4=3T/4$. Consequently, controlling the time delay T_D represents an efficient modality to manage the power transfer between two energy reservoirs. In this case the reciprocal network may consist from just one reactive element – a series inductance or a parallel capacitance – as in [5]. In some applications the reciprocal network may be replaced by a λ/4 transmission line [6]. This is a predictable result as a transmission line can be approximated by a cascade of Γ or Π *LC* two-ports.

IV. DC-DC CONVERTERS WITH FORCED GYRATOR-LIKE BEHAVIOR

By making use of a sliding-mode control, some dc-dc converters can be forced to show a dc gyrator-like behavior [11]. This idea will be illustrated with reference to the ideal buck converter shown in Fig.6, where a continuous-conduction mode will be assumed. As can be seen, the standard buck topology has been supplemented with a *LC* input filter in order to reduce the *EMI* level. In the state-space this circuit represents a 4-th order dynamical system, the state vector being $x = [i_{L1}, i_{L2}, u_{C1}, u_{C2}]^t$. Kirchhoff's laws and constitutive relations yield the state equations of the system:

$$\frac{di_{L1}}{dt} = -\frac{1}{L_1}u_{C1} + \frac{1}{L_1}U_s\,, \qquad (19)$$

$$\frac{di_{L2}}{dt} = \sigma\frac{1}{L_2}u_{C1} - \frac{1}{L_2}u_{C2}\,, \qquad (20)$$

$$\frac{du_{C1}}{dt} = \frac{1}{C_1}i_{L1} - \sigma\frac{1}{C_1}i_{L2}\,, \qquad (21)$$

$$\frac{du_{C2}}{dt} = \frac{1}{C_2}i_{L2} - \frac{1}{RC_2}u_{C2}\,, \qquad (22)$$

where σ(t) is a discrete switching variable: σ(t)=*1* if $t\in T_{on}$ (Sw closed), respectively σ(t)=*0* if $t\in T_{off}$ (Sw opened).

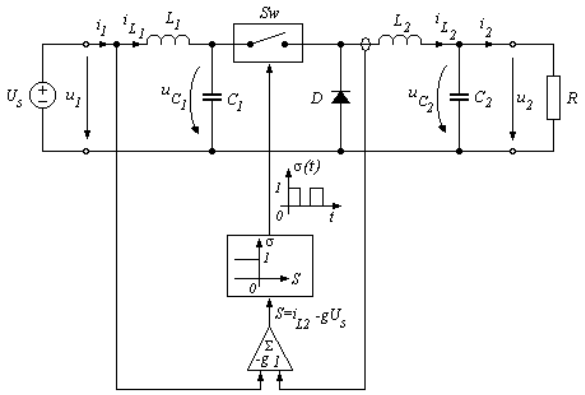

Fig.6. The buck converter with sliding-mode control feedback

If we choose the switching surface for sliding-mode control as $S = i_{L2} - gu_1$ ($u_1 = U_s$), and impose the stationarity conditions $S=0$ and $dS/dt = 0$, then eq.(20) yields

$$i_{L2} = gU_s \text{ and } \sigma = \frac{u_{C2}}{u_{C1}} \qquad (23)$$

Substituting eq.(23) in eqs.(19), (21) and (22) eventually leads to the converter dynamics under sliding-mode control:

$$\frac{di_{L1}}{dt} = -\frac{1}{L_1}u_{C1} + \frac{1}{L_1}U_s\,, \qquad (24)$$

$$\frac{du_{C1}}{dt} = \frac{1}{C_1}i_{L1} - \frac{u_{C2}}{u_{C1}}\frac{1}{C_1}gU_s\,, \qquad (25)$$

$$\frac{du_{C2}}{dt} = \frac{1}{C_2}gU_s - \frac{1}{RC_2}u_{C2}\,. \qquad (26)$$

Due to eq.(25), this is a nonlinear dynamical system. It is interesting to note, as equation (26) shows, that the dynamics of u_{C2} is not related to the rest of the state variables – it increases exponentially with a time constant RC_2 towards the steady-state value gRU_s.

The fixed point x^* of this dynamic system, which corresponds to the steady-state regime, results by equating to zero the right sides of eqs.(24)-(26):

$$\begin{aligned} x^* &= \left[i^*_{L1}, i^*_{L2}, u^*_{C1}, u^*_{C2}\right]^t = \\ &= \left[g^2RU_s, gU_s, U_s, gRU_s\right]^t. \end{aligned} \qquad (27)$$

From the port conditions

$$i_1 = i_{L1},\quad u_1 = U_s,\quad u_2 = u_{C2},\quad i_2 = u_{c2}/R \qquad (28)$$

and the control condition $i_{L2} = gU_s$, it follows for the steady-state regime that

$$I_1 = gU_2\,,\quad I_2 = gU_1\,. \qquad (29)$$

These equations correspond to an ideal dc gyrator; the missing "-"sign in the second equation is due the to active sign rule chosen this time for the output port (Fig.6). It should be noted that as 0<σ<1, from eqs.(23) and (27) follows that the gyration conductance is related to the load resistance by the condition $gR<1$.

To conclude the analysis we have to check the stability of the fixed point, that is of the steady-state behavior of the dc gyrator. To this purpose we have to compute the Jacobian matrix of the nonlinear dynamical system (24)-(26):

$$J_F\left(x^*\right) = \begin{bmatrix} \frac{\partial f_1}{\partial i_{L1}} & \frac{\partial f_1}{\partial u_{C1}} & \frac{\partial f_1}{\partial u_{C2}} \\ \frac{\partial f_2}{\partial i_{L1}} & \frac{\partial f_2}{\partial u_{C1}} & \frac{\partial f_2}{\partial u_{C2}} \\ \frac{\partial f_3}{\partial i_{L1}} & \frac{\partial f_3}{\partial u_{C1}} & \frac{\partial f_3}{\partial u_{C2}} \end{bmatrix}_{x^*}, \qquad (30)$$

where f_1, f_2 and f_3 are the corresponding right sides of eqs. (24)-(26). Solving the characteristic equation

$$\begin{aligned} &det\left(\lambda[1] - J_F\left(x^*\right)\right) = 0\,, \\ &\left(\lambda + \frac{1}{RC_2}\right)\left(\lambda^2 - \frac{g^2R}{C_1}\lambda + \frac{1}{L_1C_1}\right) = 0\,, \end{aligned} \qquad (31)$$

yields the eigenvalues

$$\lambda_1 = -\frac{1}{RC_2},\ \lambda_{2,3} = \frac{g^2R}{2C_1} \pm \sqrt{\left(\frac{g^2R}{2C_1}\right)^2 - \frac{1}{L_1C_1}}\,. \qquad (32)$$

The first eigenvalue asses the exponentially stabilization of u_{C2}, as noted before. The remaining two are positive real numbers if

$$g^2 > \frac{2}{R}\sqrt{\frac{C_1}{L_1}}\,, \qquad (32)$$

respectively complex numbers with positive real parts, for the opposite case; in each case the fixed point is unstable. The stabilization of the fixed point can be made either inserting a compensation network in the feedback loop or adding a damping network to the $L_1C_1U_s$ loop.

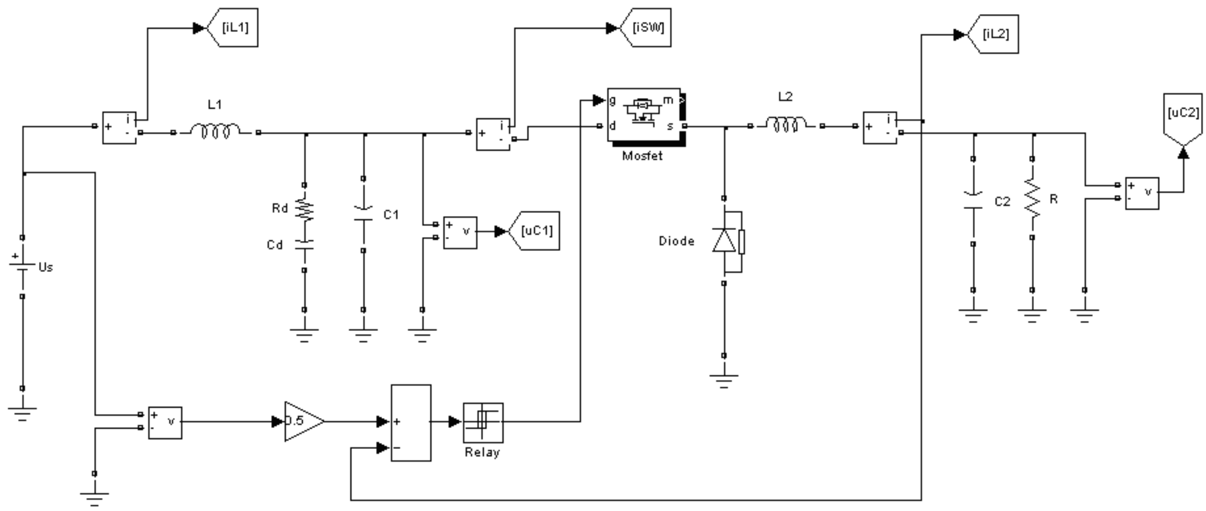

Fig.7

A simple solution for stabilizing the fixed point consists from connecting a series R_dC_d circuit in derivation with the capacitor C_1. In steady-state operation, due to the capacitor C_d, this branch is interrupted and therefore the fixed point would be the same as that of the original circuit.

The resulting 4-th order dynamical system also has the $\lambda_1 = -1/RC_2$ eigenvalue; the remaining three are the solutions of the equation

$$\lambda^3 + \frac{C_1 + C_d - g^2 R R_d C_d}{R_d C_d C_1}\lambda^2 + \left(\frac{1}{L_1 C_1} - \frac{g^2 R}{R_d C_d C_1}\right)\lambda + \frac{1}{R_d C_d L_1 C_1} = 0 . \quad (33)$$

According to Routh-Hurwitz theorem, the necessary and sufficient condition for this equation to have negative real part solutions is that the principal minors of the associated Hurwitz matrix should be positive [12]:

$$b_1 > 0,\ b_2 > 0,\ b_1 b_2 > b_3 , \quad (34)$$

where b_1-b_3 are the coefficients of λ^2 to λ^0, in this order. From eqs.(34) appropriate values for the damping RC circuit can be chosen.

To asses the gyrator-like behavior of the sliding mode controlled modified buck converter, a Simulink/SimPowerSystems model has been setup, as shown in Fig.7, where $U_s=20\ V$, $R=1\ \Omega$ and $g=0.5$. The simulation results are shown in Fig.8. As expected, the steady-state value of the output current is $gU_s=10\ A$.

Sliding-mode control can be used to force other topologies, like boost or cuk, to behave like a dc gyrator [11].

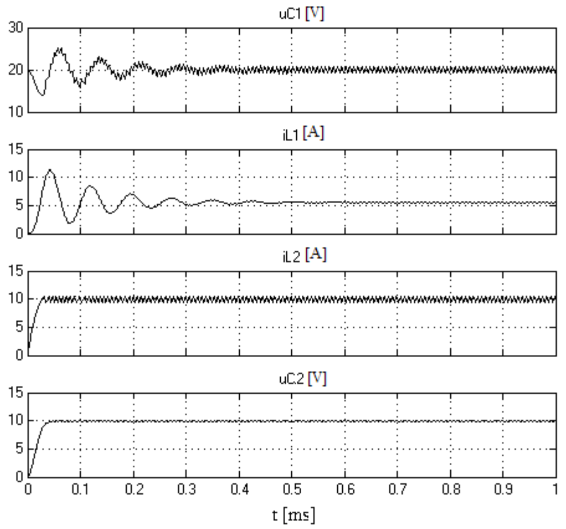

Fig.8

A possible application of gyrator-configured dc switch converters, where the controlled current source output nature of gyrators may be advantageous, is in the management of distributed power systems [11]. In this case, paralleling n gyrator-like dc converters simple means the addition of the output currents g_kU_{sk} ($k=1, 2,\ldots,n$) of the individual converters, as shown in Fig.9. However, for each individual gyrator to evolve along the imposed sliding surface, the corresponding control variable σ_k must be in the [0, 1] interval, which yields the constrain

$$0 < \frac{g_1 U_{s1} + g_2 U_{s2} + \ldots + g_n U_{sn}}{U_{sk}} R < 1 . \quad (35)$$

If all the input sources have the same value, as well as all gyration conductances, rel.(35) becomes

$$g < \frac{1}{nR} . \quad (36)$$

Different distribution strategies for the currents are possible due to the fact that each output current is independently controlled [11].

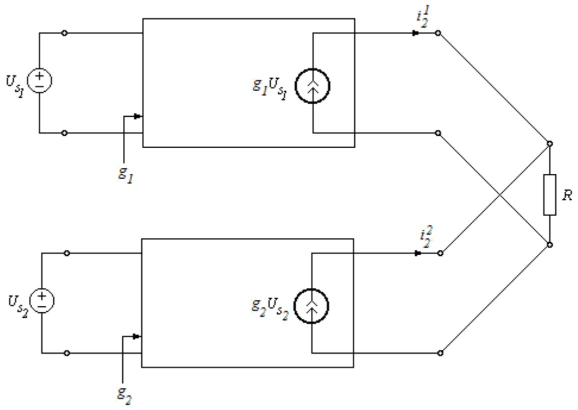

Fig.9

Although different converter topologies can be used for paralleling purposes, the buck gyrator-like converter seems to be the best choice, as demonstrated in [11].

V. CONCLUSIONS

The paper analyses two types of gyrator-like dc-dc converters which are used in dc power management. The first one, which has an unforced dc gyrator-like behavior, is operated at constant frequency. The dc power transfer from one port to the other can be simply and efficiently controlled by varying the time delay between the switched bridges. The gyrator-like behavior of the second converter type is forced with an appropriate sliding-mode control. This converter type is suited for applications implying paralleling of converters in distributed power systems, voltage regulation and impedance matching.

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